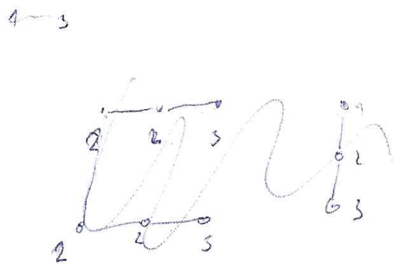
Efficient algs



Def: Clique-width

Let $k \geq 2$.

a k-labelled graph $\bar{G}$ is G with labbeling $V(G) \to \{1, \dots, k\}$.

A k-expression is a well-formed expr. made of

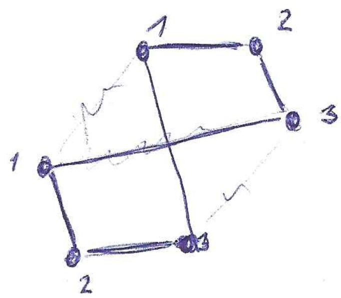
* $\bullet_i$ making a new vertex of label i,
* $\oplus$ making a disjoint union,
* $\rho_{i \to j}$ relabeling all i-s to j-s.
* $\eta_{i,j}$ adding all edges $\{u,v\}$ such that $lab(u)=i$, $lab(v)=j$, $i \neq j$.

The clique-width of G is the least k such ~~that some labeling of G is~~ the least k such that some labelling of G is built from a k-expression.

$cw(K_n) = 2 = cw(K_{n,n})$ $\quad K_{n/2} \oplus K_{n/2}$ $\eta_{1,2}$

$cw(P_n) \leq 3$ $\quad cw(C_n) \leq 4$

its not inherited on subgraphs



Prop: clique-w is monotone under induced subgraphs, but not subgraphs

THM: All MSO_1 prop can be solved in linear time on graphs of bounded cw.

Prop Hamiltonian path can be solved in time $O(n^{f(cw(G))})$ (called XP)

Proof: Compute recursively on the expression tree.

For a partial k-lab graph $\overline{H}$ $(\subseteq G)$, remember the following about a "fraction" P (= linear forest): * $\forall i, j \in \{1..k\}$, the number of paths in P having ends of the lab. i and j. $O(n^{cw^2})$

This can be processed by brute force ☒

THM: THERE EXIST MSO_2 problems which do not have XP-time alg. wrt. cw, unless $P_1 = NP_1$.

DEF: RANK-WIDTH

Vektor space

Field wrt $= \{0,1\}$

For $X \subseteq V(G)$, the cut-rank $\rho_G(X) = \mathrm{rank}\left(X \boxed{\mathrm{adj}}^{V \setminus X}\right)$ over $GF(2)$.

matrix

The rank-width $rw(G)$ = branch width ρ_G over $V(G)$.

PROP ~~$cw(G) \leq rw(G)+1 \leq 2^{cw(G}$~~

$rw(G)+1 \leq cw(G) \leq 2^{rw(G)}$

THM Rank-width is monotone under induced subgr and local complementation. (→ vertex minor)

THM: For every r-fixed, there is a cube-time alg that either finds an optimal rank-dec. of G, or confirms that $rw(G) > r$.

DEF: Labelling parse tree of G (for a rankwidth)

a k-labbeled graph $\overline{G}$ is G with $V(G) \to 2^{\{1 \dots k\}}$

i.e. $V(G) \to GF(2)^k$

a labelling join is $G \otimes H$ defined on $G \dot\cup H$

plus such edges $\{u, v\}$, $u \in V(G)$, $v \in V(H)$

that $lab(u) \cdot lab(v) = 1$.

(scalar product)

a labbeling composition $\overline{G_1} \otimes[f; g_1, g_2] \overline{G_2}$ creates the

graph $\overline{G_1} \otimes f(\overline{G_2})$ with labeling as in

$g_1(\overline{G_1}) \dot\cup g_2(\overline{G_2})$.

matrix A_f

uv edge $\Leftrightarrow$ $lab(u) \cdot A_f \cdot lab(v)^T = 1$.

a k-labeling parse tree is built of vertices and k-labeling compositions