

TREE-STRUCTURED DECOMPOSITION

def. A tree decomposition of a graph G is a pair (T, β) where T is a tree and $\beta: V(T) \rightarrow 2^{V(G)}$, such that

- $\bigcup_{x \in V(T)} \beta(x) = V(G)$
- $\forall e \in E(G) \exists x \in V(T) \text{ s.t. } e \subseteq \beta(x)$
- $\forall x, y, z \in V(T)$ s.t. y lies on the T -path between x and z it holds $\beta(y) \supseteq \beta(x) \cap \beta(z)$

interpolation property

The sets $\beta(x)$ are called bags of (T, β)



The width of (T, β) is the maximum $|\beta(x)| - 1$, $x \in V(T)$

DEF. The tree-width of G is the minimum width over all tree-decomp. of G .

PROP. G has tree-width 1 if and only if G is a forest. a simplification of G is a forest, i.e. & iff G has no K_3 -minor.

simplification
→ loops and parallel edges

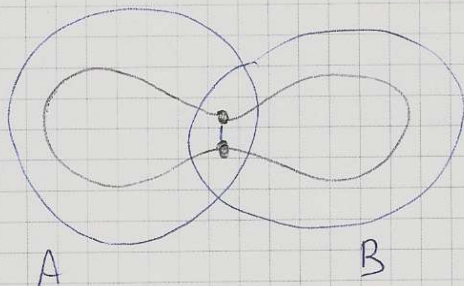
PROP. The tree-width of G is the same as that of a simplification of G .

LEMMA 3: G has tree-width ≤ 2 if and only if G has no K_4 minor.

LEMMA 4: If H is a minor of G , then $\text{tw}(H) \leq \text{tw}(G)$.
- opise trivial (constructions don't increase tw)

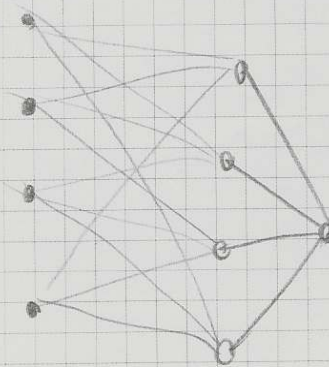
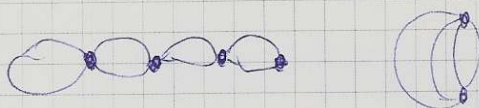
proof of LEMMA 3: $\text{Ar}(k_1)=3$, and hence \Rightarrow holds.

\Leftarrow 3-connected graphs always have a k_1 minor and so our G has a 2-cut



Make G_A and G_B (both with a new edge on the cut), and recurse.
Construct tree-decomp. of G_A, G_B of width ≤ 2 , then paste together

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