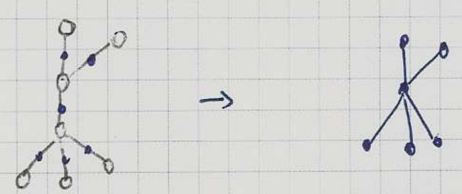
$\boxed{\text{TREE-STRUCTURED DECOMPOSITION}}$

def. A tree decomposition of a graph G is a pair (T, β) where T is a tree and $\beta : V(T) \to 2^{V(G)}$, such that

- $\bigcup_{x \in V(T)} \beta(x) = V(G)$
- $\forall e \in E(G) \; \cancel{\exists x \in (T)} \; \exists x \in V(T)$ s.t. $e \in \beta(x)$
- $\forall x, y, z \in V(T)$ s.t. y lies on the T path between x and z it holds $\beta(y) \supseteq \beta(x) \cap \beta(z)$ — interpolation property

The sets $\beta(x)$ are called bags of (T, β)



The width of (T, β) is the $\max_{x \in V(T)} |\beta(x)| - 1$.

DEF. The tree-width of G is the minimum width over all tree-decomp. of G.

PROP. G has tree-width 1 if and only if ~~G is a forest~~. a simplyfication of G is a forest, i.e. & iff G has no K_3-minor.

simplification - loops and parallell edges

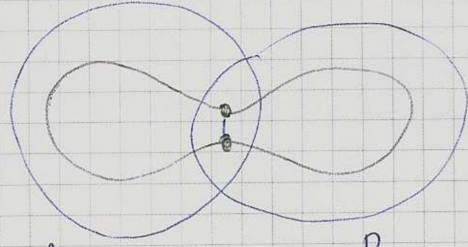
PROP. The tree-width of G is the same as that of a simplification of G.

LEMMA 3: G has tree-width ≤ 2 if and only if G has no K_4 minor.

LEMMA 4: If H is a minor of G, then $tw(H) \leq tw(G)$.

- quite trivial (contractions don't increase tw) proof of LEMMA 3: $tw(K_4)=3$, and hence $\Rightarrow$ holds.

$\Leftarrow$ 3-connected graphs always have a K_4 minor and so our G has a 2-cut



A B

Make G_A and G_B (both with a new edge on the cut), and recurs. construct tree-decomp. of G_A, G_B of width ≤ 2, then paste together

Rem. SERIES-PARALELL GRAPHS