THM : path-width $\leq k \iff k+1$ cops strategy $\iff k+1$ cops monotone strategy all for invisible robber

③ lazy robber - the robber moves only when a cop lands on him

Prop: The number of needed cops is always always between $\delta(G)+1$ and $\max(tw(G)+1, D(G)+1)$

LEM : If $tw(G) \leq k$, then $k+1$ cops have a monotone strategy against the invisible lazy robber

Proof : Just do DFS on tree-decomp ◻

(cordal - no induced cycle without cords)

*Thm : $tw(G) = \min$ of $\omega(G)-1$ (clique size) over all chord $G' \supseteq G$.

Proof : "$\Rightarrow$" Take the tree-decomp. of G ad fill evey bag into a clique.

"$\Leftarrow$" Every chordal graph contains a simplicial vertex (a vertex whose neighb. is a clique) s, then remove s, decompose $G-s$ recursively, and add a new bag containing s with its neighbours.

LEM: if $k+1$ cops have strategy against invisible lazy robber, then tree-width is at most k

Proof : order the vertices based on the last moment a cop lands on them, say as $(v_1, v_2, \dots, v_n)$

Let $B(v_i)$ be set of v_j $j<i$ such that there is a path in G from v_i to v_j using only internal vertices from $\{v_{i+1}, \dots, v_n\}$ whole set $B(v_i)$ must be occupied by cops! Now, adding all edges from v_i to $B(v_i)$, $i=1,\dots,n$, gives a cordal graph of clique-size $\leq k+1$ finally apply thm *

TREE-STRUCTURE DECOMPOSITION

- More variants on <u>cops-and robber.</u>

  ① ordinary (last time)

  + monotone

  ② invisible robber - a different story, need many cops already on a tree! ∇

<u>DEF</u>: Patwidth of a graph G is the minimum width of a tree decomposition of G such that its tree is a path

<u>LEMMA</u>: If G has path-width k, then $k+1$ cops can monotonically catch an invisible robber

<u>LEMMA</u>: If $k+1$ cops have monotone strategy to catch a invisible robber on G, then G has path-width $\leq k$.

Proof: We play the robber and "cheat" - robber is not in the graph! The cops play and we record the occupied subsets (bags) of vertices in time.

This is a path-deco: time is linear → a path wining strategy → all vertices and edges get into bags, monotonicity → interpolation prop ☐

Du ! Describe strategy for robber on complete cubic tree with height h (number of edges) with only h cops

<u>LEMMA</u>: If $k+1$ cops have a strategy, then they have a monotone strategy.

Proof is analogous to previous for a "minimal" strategy.

(minimal = cops land only on vert. where the robber may currently sit.)

From the non-interpolated decomposition, keep just last occupation interval for each vertex → interpolation ok.

(4) Ordinary game, but count number of cop moves. Then

$$\lfloor \log_2 \ell \rfloor + 1 \leq k \leq \ell + 1.$$

(corresponds tree depth)

Def (tree depth): $td(K_1) = 0$

$$td(G+H) = \max(td(G), td(H))$$

$$td(G) = \min_{x \in V(G)} td(G-x) + 1$$

Next: well quasi-ordering

large grid

big pathwidth → cubic tree