$$f_1 := \frac{1}{x}$$

$$f_2 := \frac{1}{x^2} + k \cdot \frac{1}{x}$$

$$f_2 \perp f_1 \Leftrightarrow \langle f_1, f_2 \rangle = 0 \Leftrightarrow \int_1^2 \frac{1}{x}\left(\frac{1}{x^2} + k \cdot \frac{1}{x}\right) = 0$$

$$\Leftrightarrow k = -\frac{\langle \frac{1}{x^2}, f_1 \rangle}{\langle f_1, f_1 \rangle} = -\frac{\int_1^2 \frac{1}{x^2} \cdot \frac{1}{x}\, dx}{\int_1^2 \left(\frac{1}{x}\right)^2 dx}$$

$$\langle f_1, f_1 \rangle = \int_1^2 \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_1^2 = \frac{1}{2}$$

$$\langle f_1, f_2 \rangle = \int_1^2 \frac{1}{x^3}\, dx = \left[-\frac{1}{2x^2}\right]_1^2 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

$$f_2 = \frac{1}{x^2} - \frac{3}{4x}$$

$$f_3 := x + k \cdot f_1 + l \cdot f_2$$

$$k = -\frac{\langle x, f_1 \rangle}{\langle f_1, f_1 \rangle} = -\frac{\int_1^2 1\,dx}{\frac{1}{2}} = -2$$

$$l = -\frac{\langle x, f_2 \rangle}{\langle f_2, f_2 \rangle} = -\frac{\int_1^2 \left(\frac{1}{x} - \frac{3}{4}\right) dx}{\frac{1}{96}} =$$

$$= -96 \cdot \left(\ln(2) - \frac{3}{4}\right)$$

$$= \underbrace{72 - 96\ln(2)}$$

$$\langle f_2, f_2 \rangle = \int_1^2 \left(\frac{1}{x^2} - \frac{3}{4x}\right)^2 dx$$

$$= \int_1^2 \left(\frac{1}{x^4} - \frac{3}{2x^3} + \frac{9}{16x^2}\right) dx = \cdots = \frac{1}{96}$$

5 10-14:08

$$pr\left(\frac{1}{x^3}\right) = \underbrace{\frac{\langle \frac{1}{x^3}, f_1 \rangle}{\langle f_1, f_1 \rangle}}_{\in \mathbb{R}} \cdot f_1 + \frac{\langle \frac{1}{x^3}, f_2 \rangle}{\langle f_2, f_2 \rangle} \cdot f_2$$

$$\frac{\langle \frac{1}{x^3}, f_3 \rangle}{\langle f_3, f_3 \rangle} \cdot f_3$$

Na $\langle -\pi, \pi \rangle$ uvažime podpriestor

generovaný $1, \sin x, \cos x, \sin(2x), \cos(2x), \ldots$

Veľkosť funkcií $\sin(nx)$ i $\cos(nx)$ je $\sqrt{\pi}$:

$$\langle \cos nx, \cos nx \rangle = \int_{-\pi}^{\pi} (\cos nx)^2 \, dx = \left[\begin{array}{ll} u = \cos nx & u' = -n \sin nx \\ v' = \cos nx & v = \frac{\sin nx}{n} \end{array} \right]$$

$$= \underbrace{\left[\frac{1}{n} \cos nx \sin nx \right]_{-\pi}^{\pi}}_{0} + \int_{-\pi}^{\pi} \sin^2 nx \, dx =$$

$$= \int_{-\pi}^{\pi} (1 - \cos^2 nx) \, dx = 2\pi - \int_{-\pi}^{\pi} \cos^2(nx) \, dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(nx)\, dx \qquad a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\, dx$$

Rozvoj funkce x:

$$\int_{-\pi}^{\pi} x\, dx = 0$$

$$\int x \sin(nx) =$$

$$\begin{bmatrix} u = x & u' = 1 \\ v' = \sin(nx) & v = -\frac{\cos(nx)}{n} \end{bmatrix}$$

$$= -\frac{1}{n} x \cos(nx) + \frac{1}{n} \int \cos(nx) =$$

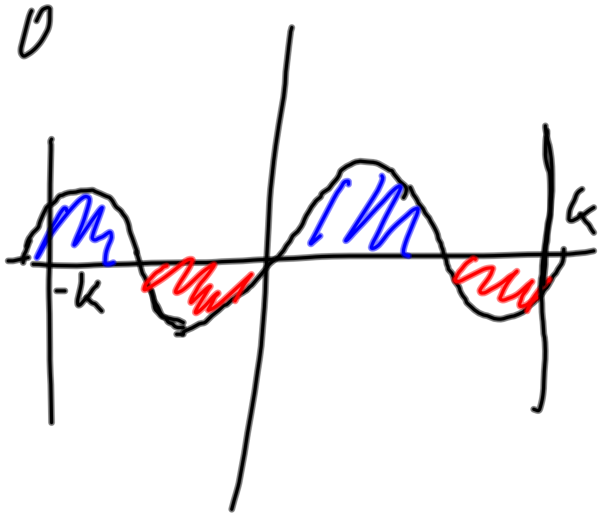
$$= -\frac{1}{n} x \cos(nx) + \frac{1}{n^2} \sin nx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx = \frac{1}{\pi} \left[-\frac{1}{n} x \cos(nx) + \frac{1}{n^2} \sin(nx) \right]_{-\pi}^{\pi} =$$

$$= -\frac{1}{\pi} \left[\frac{(-1)^n}{n} \pi + \pi \cdot \frac{(-1)^n}{n} \right] = 2 \cdot \frac{(-1)^{n+1}}{n}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \cos(nx)\,dx = 0$$

(neb $x\cos(nx)$ je lichá fce)



$$x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot \sin(nx)$$

$$\mathrm{Fr}(x+1) = \mathrm{Fr}(x) + \mathrm{Fr}(1) = 1 + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$

$$f_1 : \mathbb{R} \to \mathbb{R}, \quad f_2 : \mathbb{R} \to \mathbb{R}$$

$$f_1 * f_2 : \mathbb{R} \to \mathbb{R}$$

$$f_1 * f_2(t) = \int_{-\infty}^{\infty} f_1(x) \cdot f_2(t-x)\, dx =$$

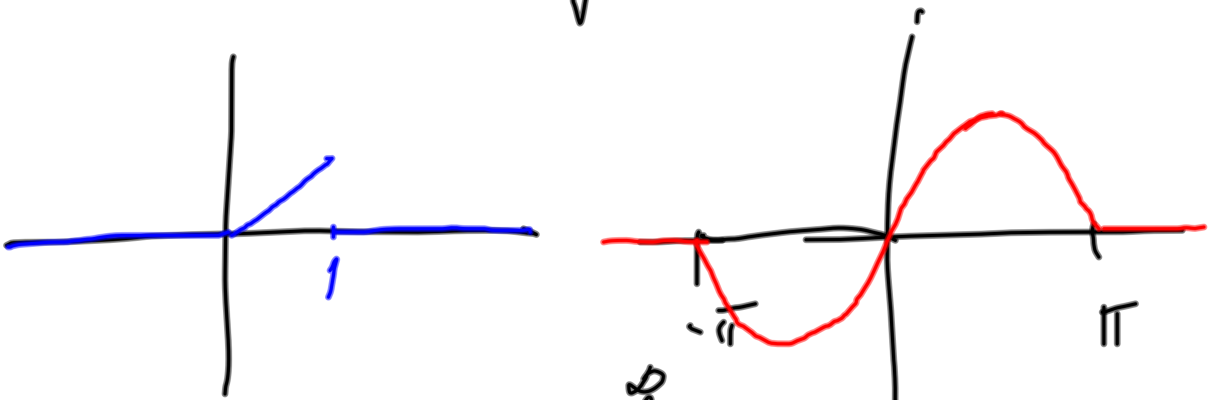
$$\begin{bmatrix} y = t - x \\ dy = -dx \end{bmatrix}$$

$$-\int_{\infty}^{-\infty} f_1(t-y) \cdot f_2(y)\, dy = \int_{-\infty}^{\infty} f_1(t-y) \cdot f_2(y)\, dy =$$

$$= f_2 * f_1(t)$$

$$f_1(x) = \begin{cases} x & \text{pro } x \in \langle 0, 1 \rangle \\ 0 & \text{jinak} \end{cases}$$

$$f_2(x) = \begin{cases} \sin x & \text{pro } x \in \langle -\pi, \pi \rangle \\ 0 & \text{jinak} \end{cases}$$



$$f_1 * f_2(t) = \int_{-\infty}^{\infty} f_1(x) \cdot f_2(t-x)\, dx$$

Hodnota funkce $f_1(x)\cdot f_2(t-x)$ může být nenulová pouze pokud $x\in\langle 0,1\rangle$ & $(t-x)\in\langle -\pi,\pi\rangle$

$$x\in\langle 0,1\rangle \Leftrightarrow t-x\in\langle t-1,t\rangle \Rightarrow$$

$$\Rightarrow \langle t-1,t\rangle\cap\langle -\pi,\pi\rangle\neq\emptyset$$

Jaký může být $\underbrace{\langle t-1,t\rangle\cap\langle -\pi,\pi\rangle}_{X}$?

a) $t\in\langle -\pi,-\pi+1\rangle$: $X=\langle -\pi,t\rangle$

b) $t\in\langle -\pi+1,\pi\rangle$: $X=\langle t-1,t\rangle$

c) $t\in\langle \pi,\pi+1\rangle$: $X=\langle t-1,\pi\rangle$

$$f_1*f_2(t)=\int_{-\infty}^{\infty} f_1(x)\cdot f_2(t-x)\,dx=\int_0^1 f_1(x)\,f_2(t-x)\,dx=\quad[y=t-x]$$

$$-\int_{s}^{s-1} f_1(s-y)\, f_2(y)\, dy = \int_{s-1}^{s} f_1(s-y)\, f_2(y)\, dy =$$

$$= \begin{cases} \displaystyle\int_{-\pi}^{s} (s-y) \sin(y)\, dy & \text{pro } s \in \langle -\pi, -\pi+1 \rangle \\ \displaystyle\int_{s-1}^{s} (s-y) \sin(y)\, dy & \text{pro } s \in \langle -\pi+1, \pi \rangle \\ \displaystyle\int_{s-1}^{\pi} (s-y) \sin y\, dy & \text{pro } s \in \langle \pi, \pi+1 \rangle \end{cases}$$

$$\int_{-\pi}^{s} (s-x) \sin x\, dx = s\int_{-\pi}^{s} \sin x\, dx - \int_{-\pi}^{s} x \sin x\, dx =$$

$$= s \cdot [-\cos x]_{-\pi}^{s} - [\sin x - x \cos x]_{-\pi}^{s} =$$

$$= d(-1-\cos d) - (\sin d - d\cos d + \pi) =$$

$$= -d - \sin(d) - \pi$$

$$\int_{d-1}^{d} (d-x)\sin x\,dx = d\left[-\cos x\right]_{d-1}^{d} + \left[x\cos x - \sin x\right]_{d-1}^{d} =$$

$$= \cdots = \sin(d-1) - \sin d + \cos(d-1)$$

$$\int_{d-1}^{\pi} (d-x)\sin x\,dx = d\left[-\cos x\right]_{d-1}^{\pi} + \left[x\cos x - \sin x\right]_{d-1}^{\pi}$$

$$= \sin(d-1) + d + \cos(d-1) - \pi$$

5 10-15:28

$$
f_1 * f_2(t) = \begin{cases} -t - \sin t - \pi, & \text{pro } t \in \langle -\pi, -\pi+1 \rangle \\ \sin(t-1) - \sin(t) + \cos(t-1), & \text{pro } t \in (-\pi+1, \pi) \\ \sin(t-1) + \cos(t-1) + t - \pi, & \text{pro } t \in \langle \pi, \pi+1) \\ 0 & \text{jinak} \end{cases}
$$