$$\bigcup_{n=1}^{\infty} \left\langle 1+\frac{1}{n}, 4-\frac{1}{n} \right\rangle = (1,4)$$

$$\bigcap_{n=1}^{\infty} \left(1-\frac{1}{n}, 4+\frac{1}{n}\right) = \langle 1,4 \rangle$$

$$A = \{x_1, \dots, x_n\} \subseteq \mathbb{R}$$

$$\mathbb{R} \setminus A = \bigcup_{i=1}^{n-1} (x_i, x_{i+1}) \cup (-\infty, x_1) \cup (x_n, \infty)$$

1, 0 je hromadným bodem:
$\varepsilon > 0$ libovolné, ex. nekonečně mnoho prvočísel větších než $\frac{1}{\varepsilon}$ $\left((\Rightarrow) \ \frac{1}{p} < \varepsilon \right)$

libovolný prvek množiny je izolovaným bodem:

$p_i \in P$

$$\frac{1}{p_l} > \frac{1}{p_i} \quad \text{pro } l < i$$

$$\frac{1}{p_l} < \frac{1}{p_i} \quad \text{pro } l > i$$

$$\delta := \min \left\{ \frac{1}{p_i} - \frac{1}{p_{i+1}}, \frac{1}{p_{i-1}} - \frac{1}{p_i} \right\},$$

potom $\left(\frac{1}{p_i} - \delta, \frac{1}{p_i} + \delta \right)$

3 8-14:08

$$\lim_{n\to\infty} \frac{1}{n^2} = 0$$

$$\lim_{n\to\infty} \frac{4n^3-2n^2+1}{3n^3-n^2+3} =$$

$$= \lim_{n\to\infty} \frac{4-\frac{2}{n}+\frac{1}{n^3}}{3-\frac{1}{n}+\frac{3}{n^3}} =$$

$$= \frac{\lim_{n\to\infty} 4 - \lim_{n\to\infty} \frac{2}{n} + \lim_{n\to\infty} \frac{1}{n^3}}{\lim_{n\to\infty} 3 - \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} \frac{3}{n^3}} =$$

$$= \frac{4-0+0}{3-0+0} = \frac{4}{3}$$

$$\lim_{n\to\infty} (a_n + b_n) =$$

$$= \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

$P(x)$ ... polynom stupně k $\quad a_k x^k + \cdots + a_1 x + a_0$

$Q(x)$ ... polynom stupně l $\quad b_l x^l + \cdots + b_0$

$$\lim_{n\to\infty} \frac{a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0}{b_l n^l + \cdots + b_0} =$$

$$= \begin{cases} 0 & l > k \\ \dfrac{a_k}{b_l} & l = k \\ \pm\infty & k > l \end{cases}$$

$x_n = (-1)^n$

$$\lim_{n\to\infty} \sqrt[n]{c} = \lim_{n\to\infty} c^{\frac{1}{n}} \overset{?}{=} c^{\lim_{n\to\infty} \frac{1}{n}}$$

jinak:

$c > 1$: posloupnost $x_n = \sqrt[n]{c}$ je klesající a $x_n > 1$ pro libovolné n.

Pro libovolnou zdola omezenou klesající posloupnost platí: $\lim_{n\to\infty} x_n = \inf\{x_n \mid n \in \mathbb{N}\}$

Pokud by $\inf\{\sqrt[n]{c} \mid n \in \mathbb{N}\} > 1$, řekněme $1+a$ $(a>0)$, pak uvažme $\varepsilon = \frac{a^2}{4}$

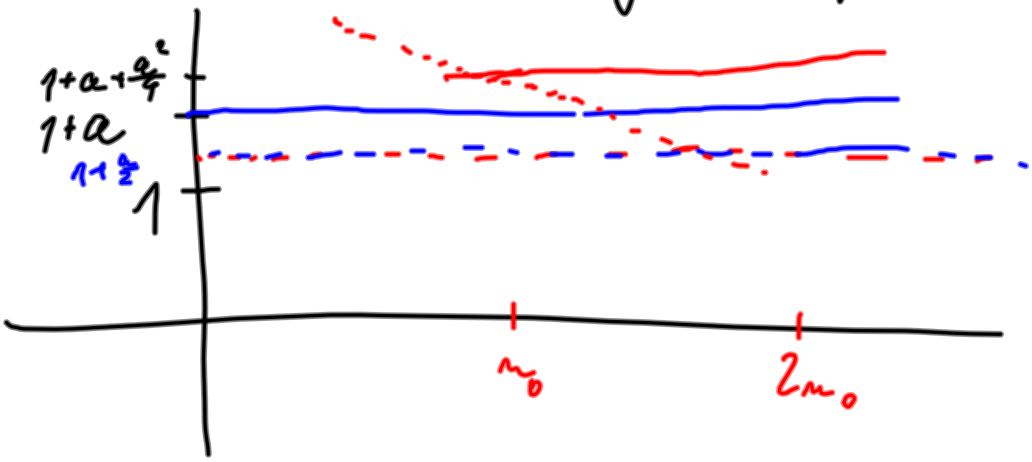
Z definice infima, existuje n_0 takové, že

$\forall n \geq n_0 \qquad x_n = \sqrt[n]{c} < 1 + a + \frac{a^2}{4}$

$$\sqrt[n]{c} < \left(1 + \frac{a}{2}\right)^2$$

$$\sqrt[2n]{c} < 1 + \frac{a}{2}$$, tedy

$1+a$ není dolní závorou, tedy ani infimem.



$$\lim_{n\to\infty} \sqrt[n]{n!} = ?$$

Nechť $k \in \mathbb{N}$ libovolné:
pak pro $n > k$ můžeme psát

$$\lim_{n\to\infty} \sqrt[n]{n!} \geq \lim_{n\to\infty} \sqrt[n]{\frac{k!}{k^k} k^n}$$

$$= \lim_{n\to\infty} k \sqrt[n]{\frac{k!}{k^k}} =$$

$$= k \lim_{n\to\infty} \sqrt[n]{\frac{k!}{k^k}} = k$$

$$\Rightarrow \lim_{n\to\infty} \sqrt[n]{n!} = \infty$$

$n \geq k$

$$n! \geq \frac{k!}{k^k} k^n$$

$$n! \geq k! \, k^{n-k}$$

$$n(n-1)\dots(k+1) \geq k^{n-k}$$

$$\lim_{x\to 4}\frac{x-4}{x^2-x-12} = \lim_{x\to 4}\frac{x-4}{(x+3)(x-4)} =$$

$$= \lim_{x\to 4}\frac{1}{x+3} = \frac{1}{7}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{1}{(\sqrt{x}+1)} = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{1}{3+2^{\frac{1}{x}}}$$

$$\lim_{x\to 0^+}\frac{1}{3+2^{\frac{1}{x}}} = \frac{1}{3+2^{\lim_{x\to 0^+}\frac{1}{x}}} = \frac{1}{3+\infty} = 0$$

$$\lim_{x\to 0^-}\frac{1}{3+2^{\frac{1}{x}}} = \frac{1}{3+2^{\lim_{x\to 0^-}\frac{1}{x}}} = \frac{1}{3+2^{-\infty}} = \frac{1}{3+0} = \frac{1}{3}$$

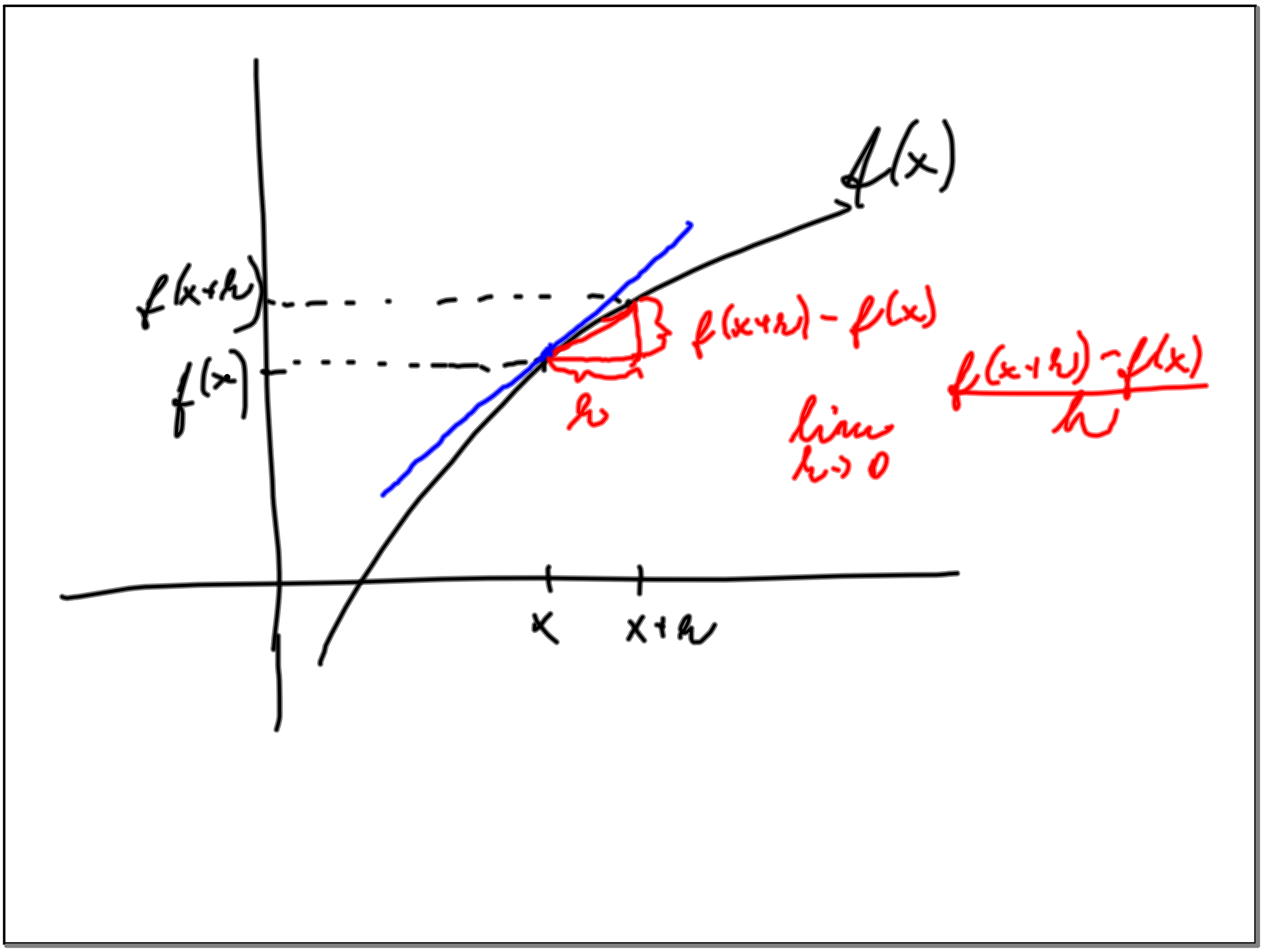
$$\lim_{x \to 2} \frac{x-2}{\sqrt{x^2-4}} = \lim_{x \to 2} \frac{x-2}{\sqrt{(x-2)(x+2)}} =$$

$$= \lim_{x \to 2} \frac{\sqrt{x-2}}{\sqrt{x+2}} = 0$$

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2} = \lim_{x \to 1} \frac{(x-1)(\sqrt{x^2+3}+2)}{x^2+3-4} =$$

$$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x^2+3}+2)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{\sqrt{x^2+3}+2}{x+1} = 2$$

3 8-15:13



3 8-15:17

$$(x^2)' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} =$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h = 2x$$

$$\lim_{x\to 0^-} \frac{\sin(x)}{x} = \frac{\sin(-x)}{-x} = \lim_{x\to 0^-} = \lim_{x\to 0^+} \frac{\sin x}{x}$$

$$\lim_{x\to 0} \frac{\sin x}{x}$$

$$S_{ABC} = \frac{1}{2}\operatorname{tg} x \geq \frac{x}{2}$$

$$\sin x \leq 1$$

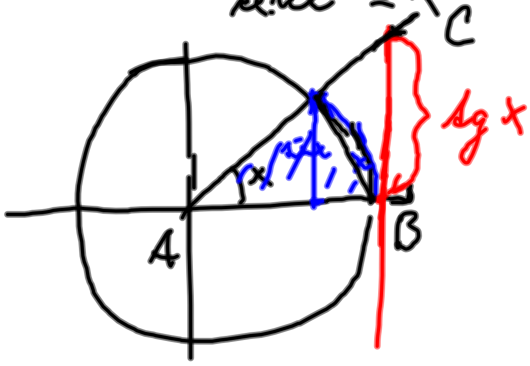
$$\operatorname{tg} x \geq x$$

$$\sin x \leq x$$

$$\lim_{x\to 0^+} \frac{\sin x}{x} \leq \lim_{x\to 0^+} \frac{x}{x} = 1$$

$$\lim_{x\to 0^+} \frac{\sin x}{x} \geq \lim_{x\to 0^+} \frac{\sin x}{\operatorname{tg} x} =$$

$$= \lim_{x\to 0^+} \cos x = 1$$



$$\left(\sin(x)\right)' = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin(x)}{h} =$$

$$= \lim_{h\to 0} \frac{\cos x \sin h}{h} + \lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} =$$

$$= \cos x + \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} = \cos x + \sin x \cdot 0 = \cos x$$

$$1 - \cos h \le 1 - \cos^2 h = \sin^2 h$$

$$\left|\lim_{h\to 0} \frac{\cos h - 1}{h}\right| = \left|\lim_{h\to 0} \frac{1-\cos h}{h}\right| \le \lim_{h\to 0} \left|\frac{\sin^2 h}{h}\right| =$$

$$= \lim_{h\to 0} h \cdot \lim_{h\to 0} \frac{\sin h}{h} = 0 \cdot 1 = 0$$

3 8-15:36