$$\lim_{n\to\infty} \frac{3^n + 2^n}{2^{2n} - 2^n} = \lim_{n\to\infty} \frac{\left(\frac{3}{4}\right)^n + \left(\frac{2}{4}\right)^n}{1 - \left(\frac{2}{4}\right)^n} =$$

$$\left(2^{2n} = \left(2^2\right)^n = 4^n \right.$$

$$= \frac{\lim\limits_{n\to\infty} \left(\frac{3}{4}\right)^n + \lim\limits_{n\to\infty} \left(\frac{1}{2}\right)^n}{1 - \lim\limits_{n\to\infty} \left(\frac{1}{2}\right)^n} = \frac{0}{1} = 0$$

$$\lim_{x\to 2} \frac{x-2}{\underbrace{\sqrt{x^2-3}}_{a} - \underbrace{1}_{b}} = \lim_{x\to 2} \frac{(x-2)\left(\sqrt{x^2-3}+1\right)}{\underbrace{x^2-3}_{a^2} - \underbrace{1}_{b^2}} =$$

$$= \lim_{x\to 2} \frac{\sqrt{x^2-3}+1}{x+2} = \frac{1}{2}$$

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$$\lim_{x\to 0} \frac{\sin(\sin(x))}{x} = \lim_{x\to 0} \frac{\sin(\sin(x))}{\sin(x)} \cdot \frac{\sin(x)}{x} =$$

$$= \underbrace{\lim_{x\to 0} \frac{\sin(\overbrace{\sin(x)}^{y})}{\underbrace{\sin(x)}_{y}}} \cdot \underbrace{\lim_{x\to 0} \frac{\sin(x)}{x}}_{1} = 1 \cdot 1 = 1$$

$$\underbrace{\lim_{y\to 0} \frac{\sin y}{y}}_{1}$$

Nechť $\lim_{t\to a} f(t) = b$.

Pokud existuje limita $\lim_{y\to b} g(y)$, pak existuje i limita $\lim_{t\to a} g(f(t))$ a jsou si rovny: (g je spojitá)

označme $\lim_{y\to b} (g(y)) = c$. tzn. $\forall\ \varepsilon > 0$, pak ex. δ takové, že $g(U_\delta(b)) \in (c-\varepsilon, c+\varepsilon)$, protože

$\lim\limits_{t\to a} f(t) = b$ tak pro δ existuje η takové, že
$f((a-\eta, a+\eta) \setminus a) \subset (b-\delta, b+\delta)$.

Př. $f(x) \begin{cases} x & \text{pro } x\in \mathbb{Q} \\ 0 & \text{pro } x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$

$g(x) = \begin{cases} 1 & \text{pro } x\in \mathbb{Q} \\ 0 & \text{pro } x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$

pak $\lim\limits_{t\to 0} f(t) = 0$

$\lim\limits_{t\to 0} g\circ f(t) = \lim\limits_{t\to 0} 1 = 1$

ale $\lim\limits_{y\to 0} g(y)$ neexistuje

$$\lim_{x\to 0} \frac{tg(x)}{arctg(x)} = \lim_{t\to 0} \frac{tg(tg(t))}{arctg(tg(t))} =$$

$x = tg\, t$

$$= \lim_{t\to 0} \frac{tg(tg(t))}{t} = \lim_{t\to 0} \frac{tg(tg(t))}{\underbrace{tg(t)}_{x}} \cdot \underbrace{\lim_{t\to 0} \frac{tg(t)}{t}}_{\lim_{t\to 0} \frac{\sin(t)}{t}\cdot\frac{1}{\cos(t)} = 1\cdot 1 = 1} =$$

$\overset{\shortparallel}{1}$

$= 1$

$$\left(\frac{1}{\sqrt{x}}\right)' = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}}{h} = \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= -\frac{1}{2x^{\frac{3}{2}}}$$

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$$(f+g)' = f' + g'$$

$$g(f(x)) = g'(f(x)) \cdot f'(x)$$

$$(f \cdot g)' = f'g + \cancel{f}g'$$

$$\left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right) = f' \cdot \left(\frac{1}{g}\right) + f \cdot \left(\frac{1}{g}\right)' = \frac{f'}{g} - \frac{fg'}{g^2} =$$

$$= \frac{f'g - fg'}{g^2}$$

$$\left(\left(\frac{x}{1+x}\right)^5\right)' = 5 \cdot \left(\frac{x}{(1+x)}\right)^4 \cdot \frac{(1+x) - x}{(1+x)^2} = 5\frac{x^4}{(1+x)^6}$$

$$x \xmapsto{f} \frac{x}{1+x} \xmapsto{g} \left(\frac{x}{1+x}\right)^5$$

$$\left(\sqrt{\frac{x^2-1}{x^2+1}}\right)' = \left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot\left(\frac{x^2-1}{x^2+1}\right)^{-\frac{1}{2}}\cdot\frac{2}{(x^2+1)^2}$$

$$\cdot 2x$$

$$\frac{x-1}{x+1} = \frac{x+1-2}{x+1} = 1-\frac{2}{x+1}$$

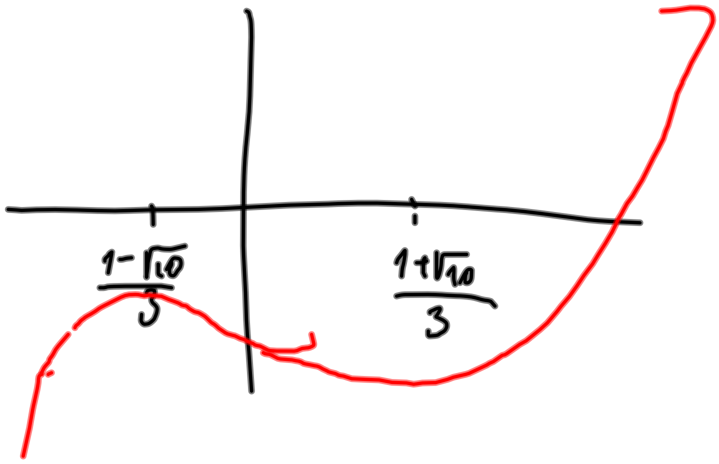
$$\left(\frac{x-1}{x+1}\right)' = \left(1-\frac{2}{x+1}\right)' = \frac{2}{(x+1)^2}$$

$$f(x) = x^3 - x^2 - 3x - 5$$

$$f'(x) = 3x^2 - 2x - 3$$

$$f'(x) = 0 \iff x_{1,2} = \frac{2 \pm \sqrt{4+36}}{6} = \frac{1 \pm \sqrt{10}}{3}$$

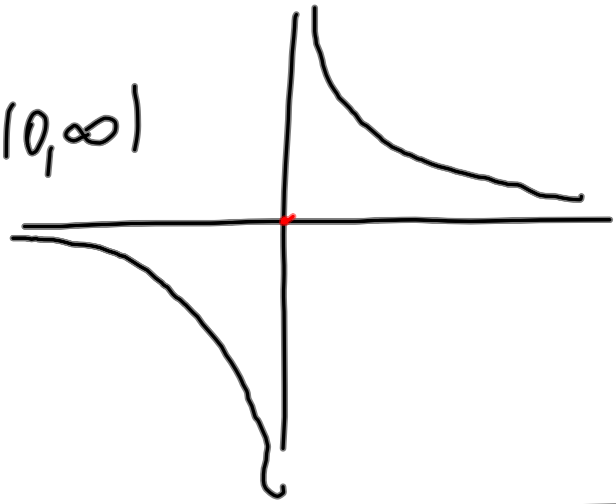
		$\frac{1-\sqrt{10}}{3}$		$\frac{1+\sqrt{10}}{3}$	
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$
$f'(x)$	$+$		$-$		$+$



$f(x) = \frac{1}{x}$ $\quad \mathcal{D}(f) = \mathbb{R} \setminus \{0\}$

$f'(x) = -\frac{1}{x^2} < 0$

Fce je klesající na int. $(-\infty, 0)$ i na int. $(0, \infty)$



$$\underbrace{f^{-1}}(f(x)) = x$$

inverzní funkce k fci f.

$$\left(f^{-1}\right)'\left(f(x)\right)\cdot f'(x) = 1$$

$$\left(f^{-1}\right)'\left(f(x)\right) = \frac{1}{f'(x)}$$

$$(\arccos)'\left(\underbrace{\cos(x)}_{y}\right) = \frac{1}{-\sin(x)} =$$

$f = \cos(x)$

$f^{-1} = \arccos(x)$

$$\left(\sin x = \sqrt{1-\cos^2 x}\right)$$

$$= -\frac{1}{\sqrt{1-\underbrace{\cos^2 x}_{y}}} \Rightarrow \arccos'(y) = -\frac{1}{\sqrt{1-y^2}}$$

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$$(\operatorname{arctg})'(\underbrace{\operatorname{tg}(x)}_{y}) = \frac{1}{\operatorname{tg}'(x)} = \frac{1}{\frac{1}{\cos^2 x}} = \cos^2 x \overset{*}{=}$$

$$(\operatorname{tg}(x))' = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$\operatorname{tg}^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x}$$

$$\cos^2 x \operatorname{tg}^2 x = 1-\cos^2 x \Rightarrow \cos^2 x = \frac{1}{1+\operatorname{tg}^2 x}$$

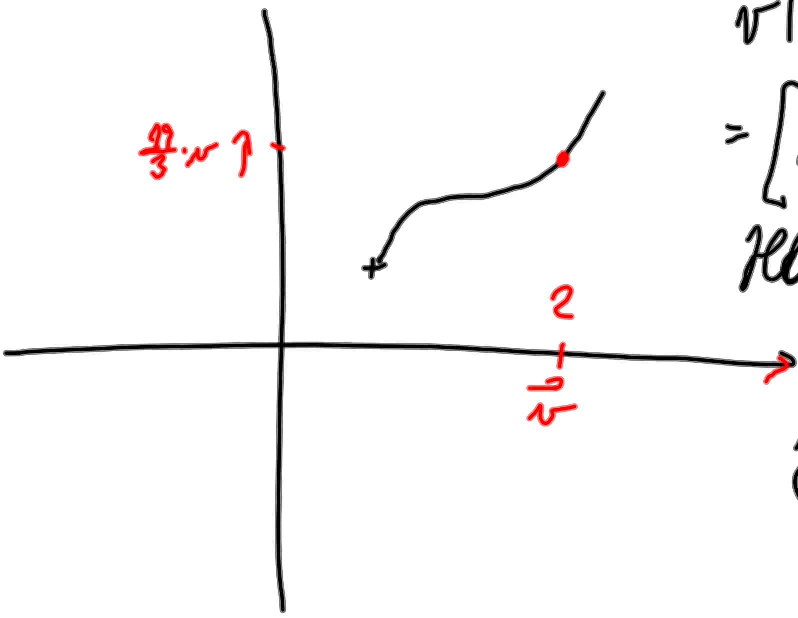
$$\overset{*}{=} \frac{1}{1+\underbrace{\operatorname{tg}^2 x}_{y}} \Rightarrow (\operatorname{arctg})'(y) = \frac{1}{1+y^2}$$

3 15-15:14

$$[t^2 - t + 1,\ t^3 + 2t^2 - t + 1] = [x(t), y(t)]$$

$$v(t) = [x'(t), y'(t)]$$

$$= [2t - 1,\ 3t^2 + 4t - 1]$$



Hledaná rychlost je $\frac{y'(t)}{x'(t)}$, pro $t = 2$ je to $\frac{19}{3}$

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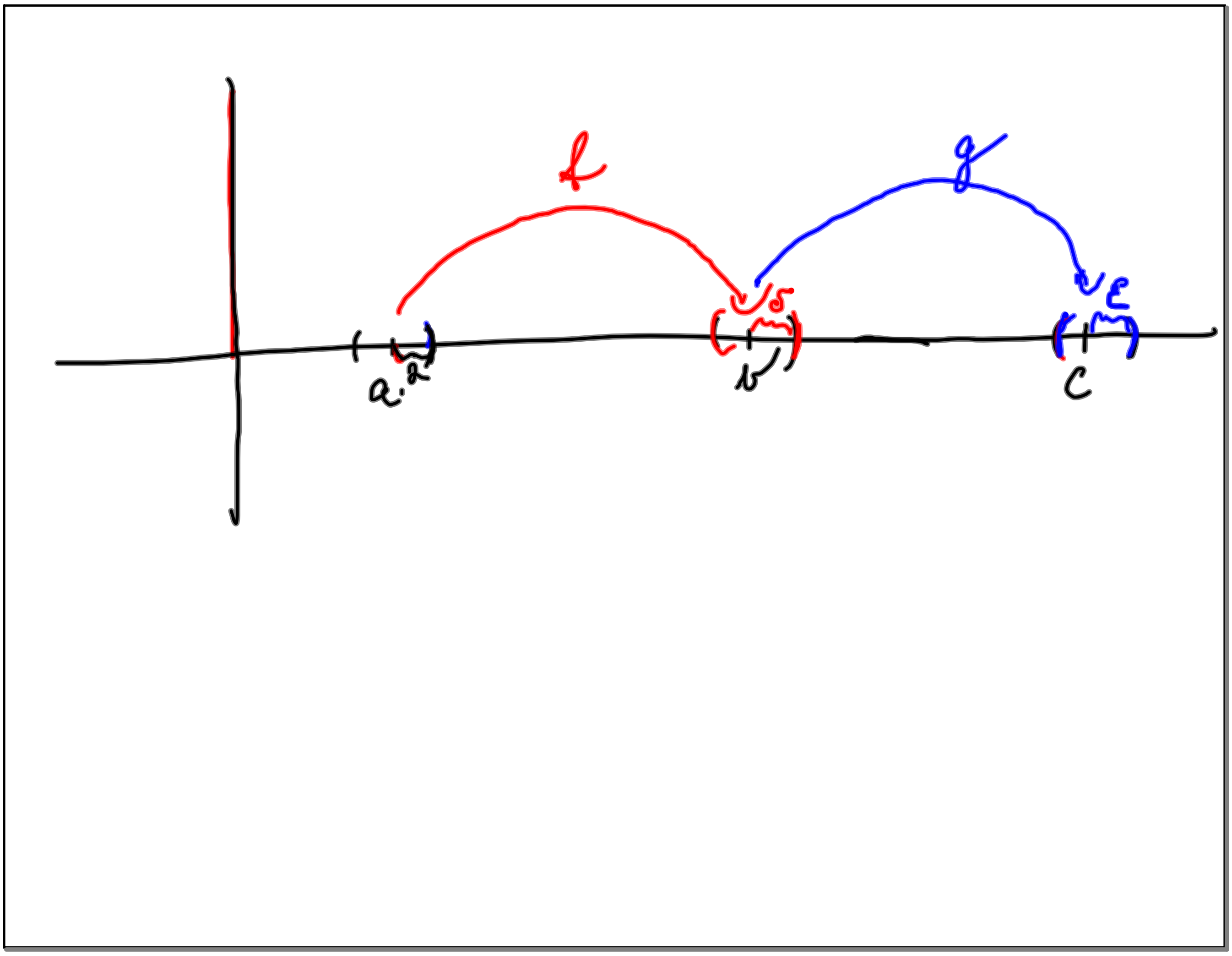

3 15-15:29