3

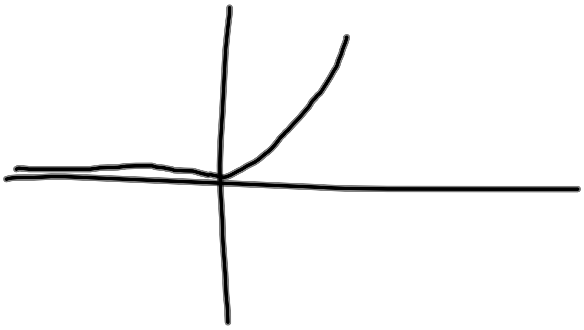
$$f(x) = \begin{cases} 0 & x < 0 \\ x^3 & x \geq 0 \end{cases}$$

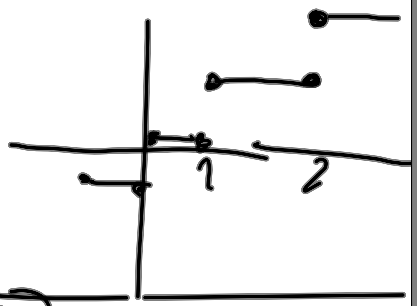
c

$$\lim_{x \to \infty} f(x) = \infty$$

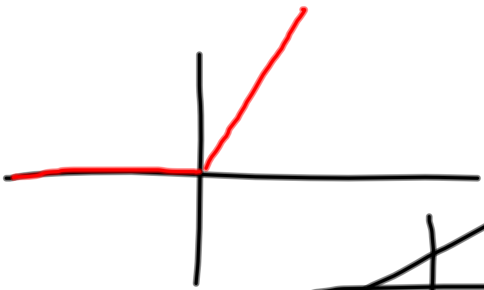
$[x]$

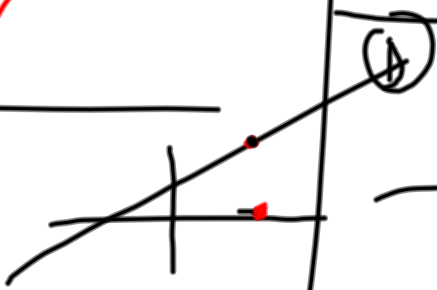
$f(x)$

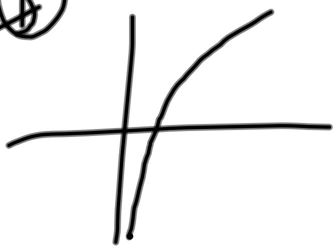




$f'(x)$







1) $\left(x^x\right)' = \left(\left(e^{\ln x}\right)^x\right)' = \left(e^{x\ln x}\right)' =$

$f(x) = x \cdot \ln x$

$\left(e^{f(x)}\right)' = f'(x) \cdot e^{f(x)} = (\ln x + 1)\, x^x$

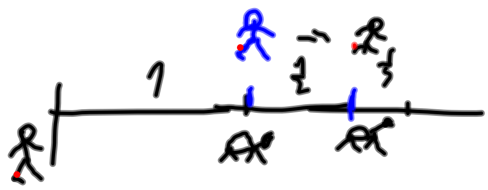
$D_f = \mathbb{R}^+$

$f'(x) = 0 \Leftrightarrow \ln x + 1 = 0 \Leftrightarrow x = \frac{1}{e}$

$\ln x + 1 < 0 \Leftrightarrow x < \frac{1}{e}$

2) $\left(x^{x^x}\right)' = \left(x^{(x^x)}\right)' = \left(e^{\ln x \cdot x^x}\right)' =$

$$= \left(\ln x \cdot x^x\right)' \left(e^{\ln x \cdot x^x}\right) = \left(\frac{x^x}{x} + \ln x \left(1 + \ln x\right) x^x\right) x^{x^x}$$



$$\sum_{n=0}^{\infty} \frac{1}{2^n} \Rightarrow$$

$$\Rightarrow \lim_{n \to \infty} \frac{\left(\frac{1}{2}\right)^{n+1} - 1}{\frac{1}{2} - 1} =$$

$$= \frac{1}{1 - \frac{1}{2}} = 2$$

$$\sum_{n=1}^{\infty} |a_n| \text{ konvergje} \Rightarrow \lim_{n \to \infty} a_n = 0$$

Ale $\underbrace{1+\frac{1}{2}}_{>\frac{1}{2}}+\underbrace{\frac{1}{3}+\frac{1}{4}}_{>\frac{1}{2}}+\underbrace{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}}_{>\frac{1}{2}}$

jde k ∞

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-$ konverguje

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ ... konverguje

Srovnávací kriterium: $\sum_{n=1}^{\infty}|a_n| = \infty$ & $|b_n| > |a_n| \Rightarrow \sum_{n=1}^{\infty}|b_n| = \infty$

podílové krit.

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = q < 1$

$\lim_{n\to\infty}\sqrt[n]{|a_n|}$

$$\lim_{n\to\infty} \frac{2^n}{n^2} = \infty \Rightarrow \sum_{n=1}^{\infty} \frac{2^n}{n^2} = \infty$$

$$\sum_{n\geq 1}^{\infty} \frac{1}{\sqrt{n}} \underset{??}{\Rightarrow} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \infty$$

$$\frac{1}{\sqrt{n}} \geq \frac{1}{n} \quad n \geq 1$$

$$\underset{a_1}{1}, \underset{a_2}{-1}, 1, -1, 1, -1$$

vechi $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^{10^5}} = c \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} = 2^{10^5} \cdot c$ ↯

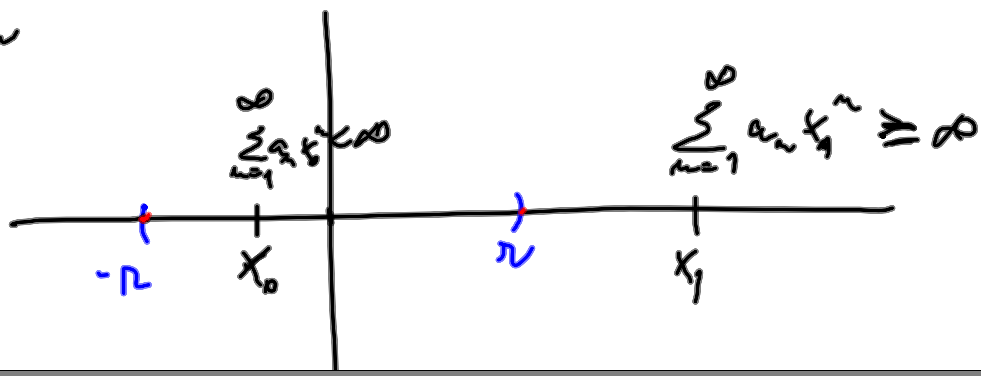
3 22-14:41

$$\sum_{n=1}^{\infty} \frac{1}{(2+i)^n} \overset{?}{=} \text{ je geometrická řada s kvocientem } q = \frac{1}{(2+i)}$$

$$|q| = \frac{1}{\sqrt{5}} < 1 \Rightarrow \text{řada konverguje}$$

$$= \frac{1}{1-\frac{1}{2+i}} = \frac{2+i}{1+i} = \frac{(2+i)(1-i)}{2} = \frac{3-i}{2}$$

$$\sum_{n=1}^{\infty} a_n x^n$$



$$\lim_{n\to\infty} \sqrt[n]{a_n x^n} = q < 1 \Rightarrow \sum_{n=1}^{\infty} a_n x^n \text{ konv.}$$

$$x \cdot \lim_{n\to\infty} \sqrt[n]{a_n} = q < 1$$

$$\Rightarrow |x| < \frac{1}{\lim_{n\to\infty} \sqrt[n]{a_n}} \Rightarrow \text{řada konverguje}$$

$$\lim_{n\to\infty} \sqrt[n]{\sqrt{\frac{1}{n}}} = \lim_{n\to\infty} \sqrt[2n]{\frac{1}{n}} = 1 \Rightarrow$$

$$\Rightarrow r = \frac{1}{1} = 1$$

$$x = -1: \quad \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$
$a_n = \frac{1}{\sqrt{n}}$ je klesající posloupn. kl. čísel.
L. K. $\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ konverguje

3 22-15:06

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^2}} = 2 \underbrace{\lim_{n\to\infty} \sqrt[n]{\frac{1}{n^2}}}_{1} = 2$$

$$\sqrt[n]{\frac{1}{n^2}} = n^{-\frac{2}{n}} = e^{-\frac{2}{n}\ln(n)}$$

$$\lim_{n\to\infty} e^{-\frac{2}{n}\ln(n)} = e^{-\lim\limits_{n\to\infty} \frac{2\ln(n)}{n}} = e^{-\lim \frac{\frac{2}{n}}{1}} = e^{0} = 1$$

$$\Rightarrow \quad \sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n \text{ konvergujú na } \left\langle \left(-\frac{1}{2}, \frac{1}{2}\right) \right\rangle$$

$$x = \frac{1}{2} : \sum_{n=1}^{\infty} \frac{1}{n^2} \ \ldots \text{ konvergujú}$$

3 22-15:17

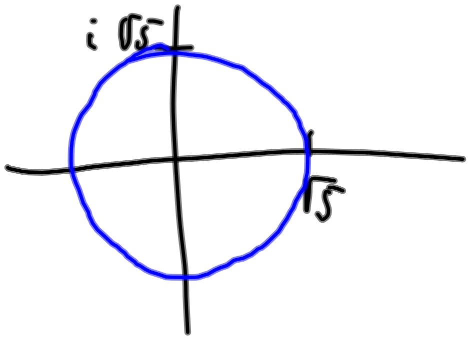
$$a_n = 1 \Rightarrow \lim_{n\to\infty} \sqrt[n]{1} = 1 \Rightarrow r = \frac{1}{1} = 1$$

$$\sum_{n=1}^{\infty} x^n \text{ konverguje právě na } (-1, 1)$$

$$a_n = \frac{1}{(2+i)^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{1}{|(2+i)|} =$$

$$\frac{1}{|(2+i)|} = \frac{1}{\sqrt{5}}$$

$$(-\sqrt{5}, \sqrt{5})$$



$$\sum_{n=1}^{\infty} x^{n!} = \sum_{n=1}^{\infty} a_n x^n \quad , \text{ kde } a_n = \begin{cases} 1 & \text{pro } n = k! \text{ pro nějaké } k \\ 0 & \text{jinak} \end{cases}$$

$\lim_{n\to\infty} \sqrt[n]{a_n}$ neexistuje

$\limsup \sqrt[n]{a_n} = 1 \qquad R = \frac{1}{1} = 1$

$\lim_{n\to\infty} \sqrt[2n+1]{\frac{1}{2n+1}} = 1 \Rightarrow R = \frac{1}{1} = 1 \qquad \langle -1, 1 \rangle$

$x = 1: \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} \xRightarrow{\text{L. k.}} \text{konverguje}$

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1} \xRightarrow{\text{L. k.}} \text{konv.}$

3 22-15:35

$$\lim_{n\to\infty} \sqrt[n]{\binom{2n}{n}} = ?$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{(2n)(2n-1)\cdots(n+1)}{n!}} \leq 2$$

---

pro jaká x konverguje $\sum_{n=1}^{\infty} \binom{2n}{n} x^n$?

podílový krit: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\binom{2n+2}{n+1}|x|}{\binom{2n}{n}} =$

$$= |x| \lim_{n\to\infty} \frac{2n+1}{n+1} = 2|x| < 1 \Leftrightarrow |x| < \frac{1}{2}$$

$$r = \frac{1}{2}$$

3 22-15:45