$$f(x) = T^{k-1}_{x_0} f(x) + \frac{1}{k!} f^{(k)}(c)(x-x_0)^k$$

$$\underbrace{f(x_0)} + \underbrace{f'(x_0)}(x-x_0) \dots + \underbrace{f^{(k-1)}(x_0)}(x-x_0)^k$$

hodnota $(k-1)$ derivace
fce f v bodě x_0

$c \in (x_0, x)$

$$\mathrm{tg}\left(\frac{\pi}{4}\right) = \underbrace{T^4_0 \,\mathrm{tg}}_{\text{polynom}}\left(\frac{\pi}{4}\right) + \frac{1}{5!}\Big(\underbrace{\mathrm{tg}^{(5)}(c)}_{\leq 512}\Big)\left(\frac{\pi}{4}\right)^5$$

$|\mathrm{tg}^{(5)}(c)| \leq 512 \quad c \in \left(0, \frac{\pi}{4}\right)$

ln je rostoucí fce na $\langle 0,\infty)$

Lemma. $f:\mathbb{R}\to\mathbb{R}$ je rostoucí & x_0 je lok. maximum fce $g:\mathbb{R}\to\mathbb{R}$. Potom x_0 je maximem fce $f\circ g$.

DŮ: x_0 je lok. max. fce g, pak v jistém prstencovém okolí $P^*_{x_0}$ platí:

$$x\in P^*_{x_0}:\ g(x)\le g(x_0) \Rightarrow f\circ g(x)\le f\circ g(x_0)$$

---

(3) $Df = (-4,-3)\cup(-2,\infty)$

Určíme extrémy fce $g(x) = \dfrac{(x+3)(x+4)}{(x+2)}$

na intervalech $[-4,-3] \cup (-2,\infty)$:

$$g'(x) = \frac{(2x+7)(x+2) - (x^2+7x+12)}{(x+2)^2} =$$

$x^2+7x+12$

$$= \frac{x^2+4x+2}{(x+2)^2}$$

$$g'(x) = 0 \,? \qquad x_{1,2} = \frac{-4 \pm \sqrt{8}}{2} = -2 \pm \sqrt{2}$$

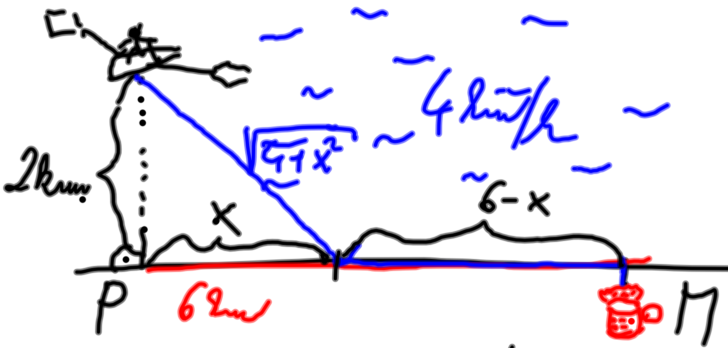
o povaze stac. bodů rozhodne nám druhá derivace

$$g''(x) = \left[\frac{x^2+4x+2}{(x+2)^2}\right]' = \left[\frac{(x+2)^2-2}{(x+2)^2}\right]' = \frac{4}{(x+2)^3} \Rightarrow g''(-2-\sqrt{2}) < 0$$

$\Rightarrow -2-\sqrt{2}$ je maximum g

$g''(-2+\sqrt{2}) > 0 \Rightarrow -2+\sqrt{2}$ je lok. minimum fce g.

Z lemmatu jsou v těchto bodech maximum resp. minimum fce $\ln\left(\frac{(x+3)(x+5)}{(x+2)}\right)$



6 km/h

$$s(x) = \frac{\sqrt{4+x^2}}{4} + \frac{6-x}{6}$$

$$s'(x) = \frac{x}{4\sqrt{x^2+4}} - \frac{1}{6} \overset{?}{=} 0$$

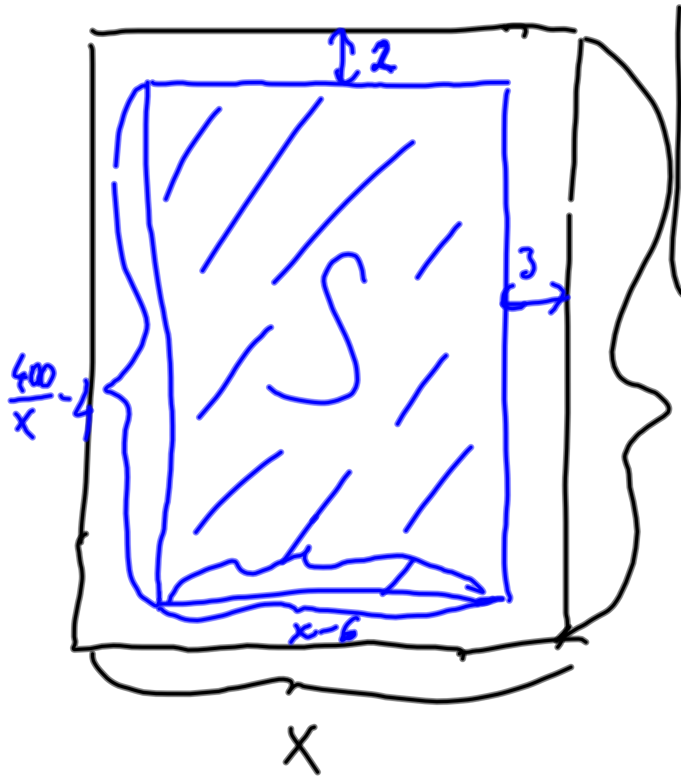
$$\frac{x}{4\sqrt{x^2+4}} - \frac{1}{6} = 0 \Rightarrow \frac{x}{4\sqrt{x^2+4}} = \frac{1}{6}$$

$$\Rightarrow \frac{x^2}{4(x^2+4)} = \frac{1}{9} \Rightarrow 9x^2 = 4x^2 + 16$$

$$s''(x) = \frac{1}{4}\,\frac{\sqrt{x^2+4} - \frac{x^2}{\sqrt{x^2+4}}}{x^2+4} = \frac{1}{4}\,\frac{4}{(x^2+4)^{\frac{3}{2}}} = \frac{1}{(x^2+4)^{\frac{3}{2}}} > 0$$

Cesta bude pro $x = \frac{4}{\sqrt{5}}$ nejrychlejší.

$x = \frac{4}{\sqrt{5}}$



$$S = (x-6)\left(\frac{400}{x}-4\right)$$

$$S' = \left[400 + 24 - 4x - \frac{2400}{x}\right]'$$

$$= -4 + \frac{2400}{x^2} = 0$$

$$\Rightarrow x^2 = 600 \Rightarrow$$

$$\Rightarrow x = 10\sqrt{6}$$

$$S'' = -\frac{4800}{x^3}$$

$\Rightarrow$ obsah tiskové plochy je maximální pro rozměry $10\sqrt{6} \times \frac{40}{\sqrt{6}}$

$D_f = \mathbb{R} - \{-1\}$

nulové body: $\frac{-1 \pm \sqrt{5}}{2}$

$$f'(x) = \frac{(2x+1)(x+1) - (x^2+x-1)}{(x+1)^2} = \frac{x^2+2x+2}{(x+1)^2} =$$

$$= \frac{(x+1)^2+1}{(x+1)^2} = 1 + \frac{1}{(x+1)^2}$$

$\Rightarrow f'(x)$ je kladná na celém svém def. oboru

		-1	
f'	>0		>0
f	$\nearrow$		$\nearrow$
f''	>0		<0
	$\cup$		$\cap$

$$f''(x) = -\frac{2}{(x+1)^3}$$

asymptoty bez směrnice.

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{x^2+x-1}{x+1} = \frac{\lim_{x\to -1^+} x^2+x-1}{\lim_{x\to -1^+} x+1} =$$

$$= \frac{-1}{0^+} = -\infty$$

$$\lim_{x\to -1^-} f(x) = \infty$$

asymptotou bez směrnice je přímka $x=-1$

Zkusme najít asymptoty se směrnicí:

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2+x-1}{x(x+1)} = 1$$

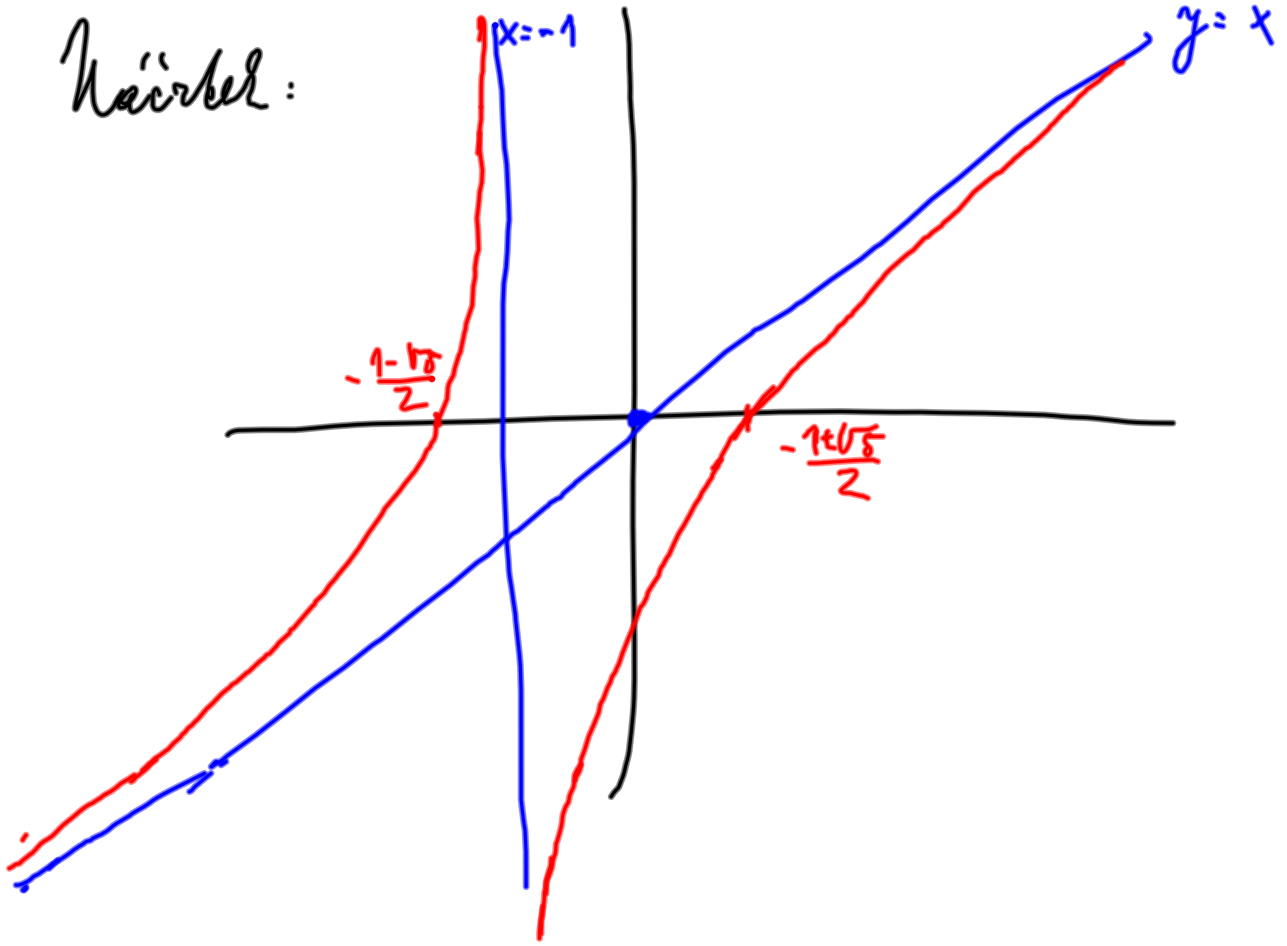
$$\lim_{x\to\infty} \left[f(x) - x\right] = \lim_{x\to\infty} \frac{x^2+x-1}{x+1} - x = \lim_{x\to\infty} \frac{x^2+x-1-x(x+1)}{x+1} =$$

$$= \frac{-1}{\infty} = 0$$

$\Rightarrow$ asymptotou se směrnicí je přímka $y=x$

4 5-15:19

Náčrtek:



$D_f = \mathbb{R} \setminus \langle -1, 1 \rangle$

nulové body: $x^2 - 1 = 1 \Rightarrow x = \pm\sqrt{2}$

funkce je sudá $(f(x) = f(-x))$

$f'(x) = \dfrac{2x}{x^2 - 1}$

f': <0 $\quad -1 \quad 1 \quad$ >0

$\searrow \qquad\qquad \nearrow$

f'': $<0 \qquad\qquad <0$

$$f''(x) \; \frac{2(x^2-1) - 4x^2}{(x^2-1)^2} = \frac{-2x^2 - 2}{(x^2-1)^2} < 0$$

asymptoty bez směrnice

$$\lim_{x \to -1^-} \ln(x^2 - 1) = -\infty \qquad \lim_{x \to 1^+} \ln(x^2-1) = -\infty$$

asymptoty se směrnicí

$$\lim_{x\to\infty} \frac{\ln(x^2-1)}{x} = \lim_{x \to \infty} \frac{\frac{2x}{x^2-1}}{1} = 0$$

$$\lim_{x \to \infty} \left(\ln(x^2-1) - 0 \cdot x \right) = \infty \Rightarrow$$

$\Rightarrow$ asymptota se směrnicí neexistuje

náčrtek