$$V = \frac{1}{3}\pi r^2 v \quad \Rightarrow \quad v = \frac{3}{\pi r^2}$$

$$P = \pi r \sqrt{r^2 + v^2} = \pi r \sqrt{r^2 + \frac{9}{\pi^2 r^4}} = \pi \sqrt{r^4 + \frac{9}{\pi^2 r^2}}$$

$$P' = \pi \frac{4r^3 - \frac{18}{\pi^2 r^3}}{2\sqrt{r^4 + \frac{9}{\pi^2 r^2}}} = 0 \Leftrightarrow$$

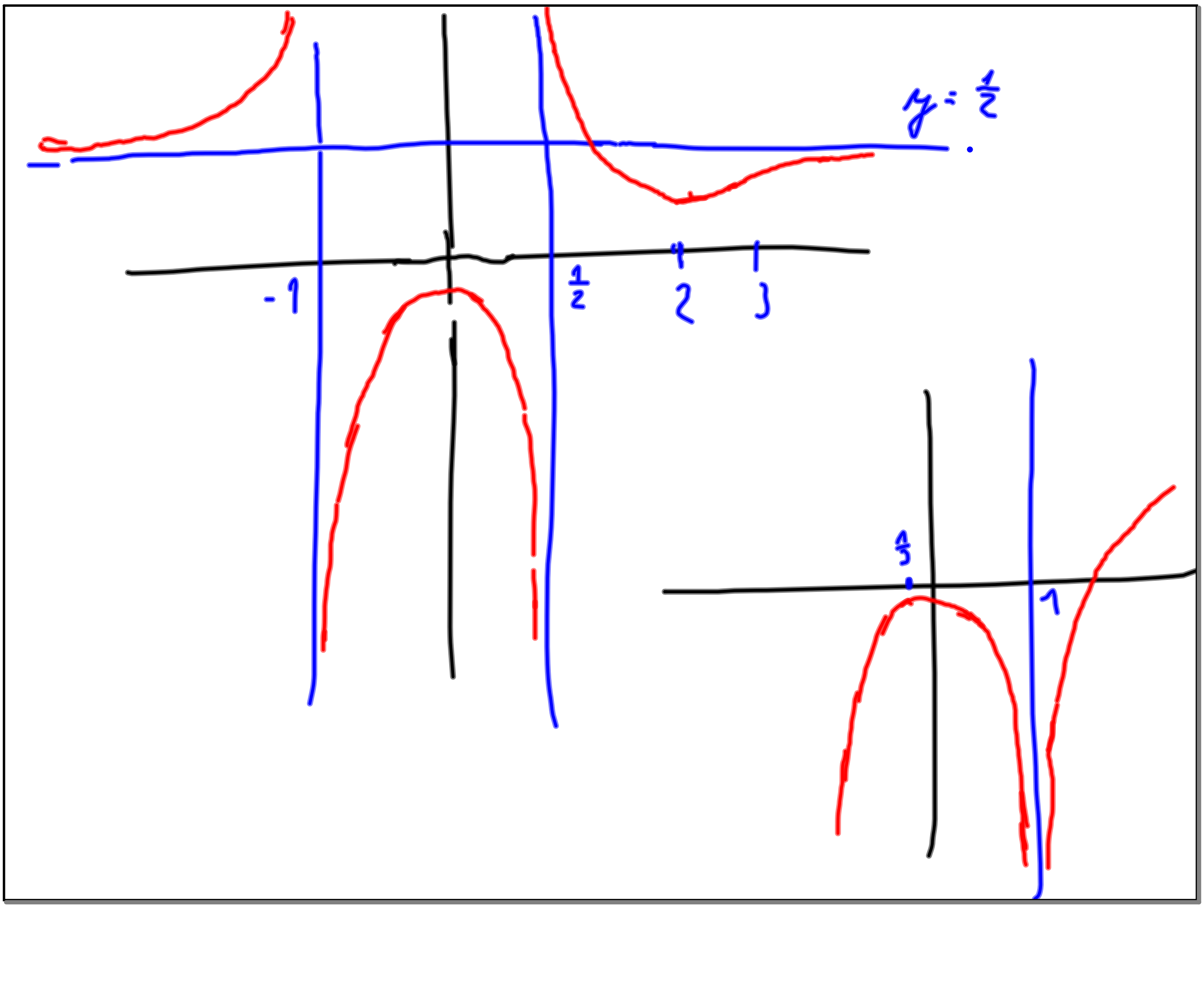
$$\Leftrightarrow \quad 4r^3 - \frac{18}{\pi^2 r^3} = 0 \Rightarrow r = \sqrt[6]{\frac{9}{2\pi^2}} \doteq 0{,}877 \text{ dm}$$

$$f''(x) = 0 \Leftrightarrow 2x^3 - 6x^2 - 1 = 0$$

$$2x^2(x-3) = 1$$

$\Rightarrow$ pro jediný reálný kořen platí

$$3 < x_0 < 3 + \frac{1}{18}$$



y = 1/2
-1
1/2
4
2
1
3
1/3
1

$$\int \operatorname{tg} x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\int \frac{1}{t} \, dt =$$

$$\begin{bmatrix} t = \cos x \\ dt = -\sin x \, dx \end{bmatrix}$$

$$= -\ln|t| = -\ln|\cos x| + C$$

$$\underline{\int \sin^2 x \, dx} = -\sin x \cos x + \int \cos^2 x \, dx =$$

$$\begin{bmatrix} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{bmatrix}$$

$$= -\sin x \cos x + \int (1 - \sin^2 x) \, dx =$$

$$= -\sin x \cos x + x - \int \sin^2 x \, dx$$

$$\Rightarrow \int \sin^2 x = \frac{1}{2}\left[x - \sin x \cos x\right]$$

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$$I_1 = \int \sin^3 x \, dx = -\cos x \sin^2 x + 2\int \sin x \cos^2 x \, dx =$$

$$\left[\begin{array}{ll} u = \sin^2 x & u' = 2\sin x \cos x \\ v' = \sin x & v = -\cos x \end{array}\right.$$

$$= -\cos x \sin^2 x + 2\int \sin x \,(1 - \sin^2 x)\, dx =$$

$$= -\cos x \sin^2 x - 2\cos x - 2\int \sin^3 x \, dx + C$$

$$\Rightarrow \int \sin^3 x = -\frac{1}{3}\cos x \sin^2 x - \frac{2}{3}\cos x + C$$

$$I_2 = \int \sin^3 x \, dx = \int \sin x (1 - \cos^2 x)\, dx = -\int (1 - t^2)\, dt =$$

$$\left[\begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array}\right]$$

$$= -\left(t - \frac{t^3}{3}\right) = \frac{\cos^3 x}{3} - \cos x$$

$$I_2 - I_1 = \frac{\cos^3 x}{3} - \cos x + \frac{1}{3}\cos \sin^2 x + \frac{2}{3}\cos x = 0$$

$$\int \arcsin x \, dx = x \cdot \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, dx =$$

$$\begin{bmatrix} u = \arcsin x & u' = \frac{1}{\sqrt{1-x^2}} \\ v' = 1 & v = x \end{bmatrix} \qquad \begin{bmatrix} t = x^2 \\ dt = 2x\,dx \end{bmatrix}$$

$$= x \cdot \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-t}} \, dt =$$

$$= x \cdot \arcsin x + \sqrt{1-t} = x \arcsin x + \sqrt{1-x^2}$$

$$\int \sqrt{1-x^2} \, dx = \int \sqrt{1-\sin^2 t} \cos t \, dt = \int \cos^2 t \, dt =$$

$$x = \sin t$$
$$dx = \cos t \, dt$$

$$= \frac{1}{2} \left[t + \sin t \cos t \right] + C$$

$$= \frac{1}{2} \left[\arcsin x + x \sqrt{1-x^2} \right] + C$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \cdot \ln x - \int \frac{x^2}{3} = \frac{x^3}{3} \cdot \ln x - \frac{x^3}{9}$$

$$\left[\begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{array}\right]$$

$$\int x \cdot \sqrt[3]{x+2} \, dx = \frac{3}{4} x (x+2)^{\frac{4}{3}} - \frac{3}{4} \int (x+2)^{\frac{4}{3}} dx =$$

$$\left[\begin{array}{ll} u = x & u' = 1 \\ v' = \sqrt[3]{x+2} & v = \frac{3}{4}(x+2)^{\frac{4}{3}} \end{array}\right]$$

$$= \frac{3}{4} x (x+2)^{\frac{4}{3}} - \frac{9}{28} (x+2)^{\frac{7}{3}} =$$

$$= (x+2)^{\frac{4}{3}} \left(\frac{3}{4} x - \frac{9}{28}(x+2)\right) = (x+2)^{\frac{4}{3}} \left(\frac{3}{7} x - \frac{9}{14}\right)$$

$$\int \frac{2x}{x^2-4x+3}\,dx = \int \frac{2x}{(x-3)(x-1)}\,dx = \int \frac{3}{x-3}\,dx - \int \frac{dx}{x-1}$$

$$\left[\begin{aligned} &\frac{2x}{(x-3)(x-1)} = \frac{A}{(x-3)} + \frac{B}{(x-1)} = \frac{3}{(x-3)} - \frac{1}{x-1} \\ &\quad 2x = A(x-1) + B(x-3) \\ &\qquad 2 = -2B \Rightarrow B = -1 \\ &\qquad 6 = 2A \Rightarrow A = 3 \end{aligned}\right]$$

$$= 3\ln|x-3| - \ln|x-1| + C$$

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$$\int \frac{x^4}{x^3+x^2+x+1}\,dx = \int (x-1)\,dx + \int \frac{1}{x^3+x^2+x+1}\,dx =$$

$$x^4 : x^3+x^2+x+1 = x-1$$
$$-(x^4+x^3+x^2+x)$$
$$-x^3-x^2-x$$
$$-(-x^3-x^2-x-1)$$
$$1$$

$$= \frac{x^2}{2} - x + \frac{1}{2}\int \frac{1}{(x+1)}\,dx - \frac{1}{2}\int \frac{x-1}{(x^2+1)}\,dx$$

$$\frac{1}{x^3+x^2+x+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{1}{2(x+1)} - \frac{x-1}{2(x^2+1)}$$
$$1 = A(x^2+1) + (Bx+C)(x+1)$$
$$1 = 2A \Rightarrow A = \frac{1}{2}$$
$$x^0:\ 1 = A + C \Rightarrow C = \frac{1}{2}$$
$$x^2:\ 0 = A + B \Rightarrow B = -\frac{1}{2}$$

$$\frac{x^2}{2} - x + \frac{1}{2}\ln|x+1| - \frac{1}{4}\int \frac{2x}{x^2+1}\,dx + \frac{1}{2}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{x^2}{2} - x + \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1)$$

$$+ \frac{1}{2}\operatorname{arctg} x .$$

$$\int \frac{f'(x)}{f(x)}\,dx = \begin{bmatrix} s = f(x) \\ ds = f'(x)\,dx \end{bmatrix} = \int \frac{1}{s}\,ds = \ln|f(x)|$$

$$\int \frac{1}{(x^2+1)^2}\,dx$$

$$(\operatorname{arctg} x)' = \frac{1}{x^2+1}$$

$$\operatorname{arctg} x = \int \frac{1}{x^2+1}\,dx = \frac{x}{x^2+1} + \int \frac{2x^2}{(x^2+1)^2}\,dx$$

$$\begin{bmatrix} u = \frac{1}{x^2+1} & u' = -\frac{2x}{(x^2+1)^2} \\ v' = 1 & v = x \end{bmatrix}$$

$$= \frac{x}{x^2+1} + 2\int \frac{x^2+1}{(x^2+1)^2} - 2\int \frac{1}{(x^2+1)^2}\,dx =$$

$$= \frac{x}{x^2+1} + 2\,\mathrm{arctg}\,x - 2\int \frac{1}{(x^2+1)^2}\,dx = \mathrm{arctg}\,x$$

$$\Rightarrow \int \frac{1}{(x^2+1)^2}\,dx = \frac{1}{2}\left(\frac{x}{x^2+1} + \mathrm{arctg}\,x\right)$$

$$\int \frac{x+1}{x^2+x+3}\,dx = \frac{1}{2}\int \frac{(2x+1)\,dx}{x^2+x+3} + \frac{1}{2}\int \frac{1}{x^2+x+3}\,dx$$

$$= \frac{1}{2}\ln(x^2+x+3) + \frac{1}{2}\int \frac{1}{(x+\frac{1}{2})^2+\frac{11}{4}}\,dx =$$

$$= \frac{1}{2}\ln(x^2+x+3) + \frac{2}{11}\int \frac{1}{\frac{4}{11}(x+\frac{1}{2})^2+1}\,dx =$$

$$\left[y = \frac{2}{\sqrt{11}}(x+\tfrac{1}{2}),\ dy = \frac{2}{\sqrt{11}}\,dx\right]$$

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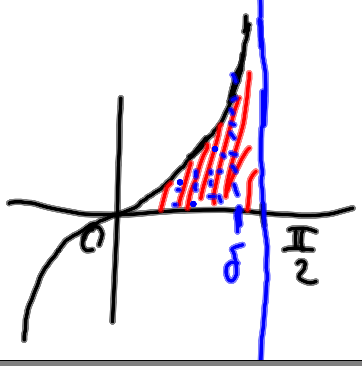
$$= \frac{1}{2}\ln(x^2+x+3) + \frac{1}{\sqrt{11}}\int \frac{1}{y^2+1}\,dy =$$

$$= \frac{1}{2}\ln(x^2+x+3) + \frac{1}{\sqrt{11}}\,\mathrm{arctg}\,y + C =$$

$$= \frac{1}{2}\ln(x^2+x+3) + \frac{1}{\sqrt{11}}\,\mathrm{arctg}\left(\frac{2}{\sqrt{11}}\left(x+\frac{1}{2}\right)\right) + C$$

$$\int_0^{\frac{\pi}{2}} \mathrm{tg}\,x\,dx = \lim_{\delta\to\frac{\pi}{2}^-} \int_0^{\delta} \mathrm{tg}\,x\,dx =$$



$$= \lim_{\delta\to\frac{\pi}{2}^-} \Big[-\ln|\cos x|\Big]_0^{\delta} =$$

$$= \lim_{\delta\to\frac{\pi}{2}^-} \left(-\ln|\cos\delta|\right) = \infty$$

$$\int_1^2 \frac{1}{\sqrt{x-1}}\,dx = \lim_{\delta \to 1^+} \int_\delta^2 \frac{1}{\sqrt{x-1}}\,dx =$$

$$= \lim_{\delta \to 1^+} \left[2\sqrt{x-1} \right]_\delta^2 = \lim_{\delta \to 1^+} \left(2 - 2\sqrt{\delta - 1} \right) =$$

$$= 2$$