$$\int \underbrace{x}_{u} \cdot \underbrace{\sin x}_{v'} \, dx = \left[uv - \int u'v + C \right] =$$

$$\begin{bmatrix} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{bmatrix}$$

$$= -x\cos x - \int -\cos x \, dx + C =$$

$$= -x\cos x + \sin x + C$$

Kdybych zvolil obráceně:

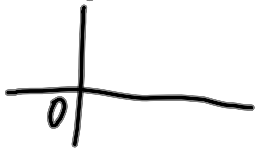
$$\begin{bmatrix} u = \sin x & u' = \cos x \\ v' = x & v = \frac{x^2}{2} \end{bmatrix}$$

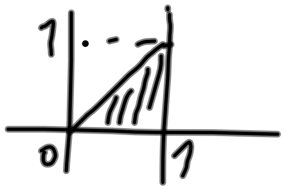
$$\int x \sin x \, dx = \frac{x^2}{2} \cdot \sin x - \int \frac{x^2}{2} \cos x \, dx =$$

$$= \ldots$$

$$\int 1 \ln x = x\ln x - \int dx = x \ln x - x + C$$

$$\left[\begin{matrix} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{matrix}\right]$$





$$\int x \, dx = \frac{x^2}{2} + C$$

$$\left[\begin{matrix} x = \sin t \\ dx = \cos t \, dt \end{matrix}\right]$$

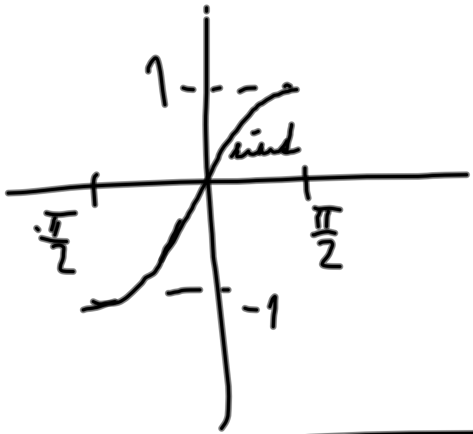
$$\int_0^1 x\,dx = \int_0^{\frac{\pi}{2}} \sin t \cos t \, dt =$$

$$\left[\begin{matrix} x = \sin t \\ dx = \cos t \, dt \end{matrix}\right]$$

$$\int x\,dx = \int \sin t \cos t \, dt = \frac{1}{2}\int \sin(2t)\,dt =$$

$$= -\frac{1}{4}\cos(2t) + c$$

$$= \left[-\frac{1}{4}\cos(2t)\right]_0^{\frac{\pi}{2}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$



$$\int \cos^{n-1} x \cos x \, dx = \ldots$$

$$\left[\begin{array}{ll} u = \cos^{n-1} & u' = -(n-1)\cos^{n-2} x \sin x \\ v' = \cos x & v = \sin x \end{array}\right]$$

$$\int \cos^2 x\,dx = \cos x \sin x + \int \sin^2 x\,dx =$$

$$\left[\begin{matrix} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{matrix}\right] \qquad \left[\begin{matrix} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{matrix}\right]$$

$$= \cos x \sin x$$

$$+ \left(-\cos x \sin x + \int \cos^2 x\,dx\right.$$

$$= \int \cos^2 x\,dx$$

$$= \cos x \sin x + \int (1 - \cos^2 x)\,dx =$$

$$= \cos x \sin x + x - \int \cos^2 x\,dx$$

$$\int \cos^2 x\,dx = \frac{1}{2}\left(x + \cos x \sin x\right) + C$$

$$\int \frac{f}{g}\,dx = \int q + \frac{h}{g}\,dx$$

$$\mathrm{gd}(h) < \mathrm{gd}\, g$$

$$(x^2+3x+2) = (x+1)(x+2)$$

$$\frac{4x+2}{(x^2+3x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} = \frac{-2}{x+1} + \frac{6}{x+2}$$

$$4x+2 = A(x+2) + B(x+1)$$

$$-6 = -B \quad | \quad A = -2$$

$$B = 6 \quad | \quad A = -2$$

$$\int \left(-\frac{2}{x+1} + \frac{6}{x+2}\right) dx$$

$$= \int -2\ln|x+1| + 6\ln|x+2| + C$$

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$$1 = (x^2+x) + x^2$$

$$\int \frac{2x^2+x}{x^3+x^2+x+1}\,dx = \int \frac{2x^2+x}{(x+1)(x^2+1)}\,dx =$$

$$\left[\begin{array}{l} x^4-1 = (x-1)(x^3+x^2+x+1) \\ x^4-1 = (x^2+1)(x^2-1) = (x^2+1)(x+1)\cancel{(x-1)} \end{array}\right]$$

$$= \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| = \frac{1}{2}\ln(x^2+1)$$

$$= \int\left(\frac{x}{x^2+1} + \frac{x^2}{(x+1)(x^2+1)}\right)dx = \frac{1}{2}\int\frac{2x}{x^2+1}\,dx + \int\frac{x^2\,dx}{(x+1)(x^2+1)}$$

$$\left[\begin{array}{l} t = x^2+1 \\ dt = 2x\,dx \end{array}\right]$$

$$= \frac{1}{2}\ln(x^2+1) \; +$$

$$\frac{x^2}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{x^2+1}$$

$$x^2 = A(x^2+1) + (Bx+C)(x+1)$$

$x=-1$: $1 = 2A \Rightarrow A = \frac{1}{2}$

$x^0: \; 0 = A + C \Rightarrow C = -\frac{1}{2}$

$x^2: \; 1 = A + B \Rightarrow B = \frac{1}{2}$

$$\int \frac{x^2}{(x+1)(x^2+1)}\,dx = \frac{1}{2}\int \frac{1}{(x+1)}\,dx + \frac{1}{2}\int \frac{x-1}{x^2+1}\,dx =$$

$$= \frac{1}{2}\ln|x+1| + \frac{1}{2}\int \frac{x}{x^2+1}\,dx - \frac{1}{2}\int \frac{1}{x^2+1}\,dx =$$

$$= \frac{1}{2}\ln|x+1| + \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\operatorname{arctg} x + C$$

$$= \frac{1}{2}\ln(x^2+1) + \frac{1}{2}\ln|x+1| + \frac{1}{4}\ln(x^2+1)\ \frac{1}{2}\operatorname{arctg} x \mp C$$

$$= \frac{3}{4}\ln(x^2+1| + \frac{1}{2}\ln|x+1)\ \frac{1}{2}\operatorname{arctg} x + C$$

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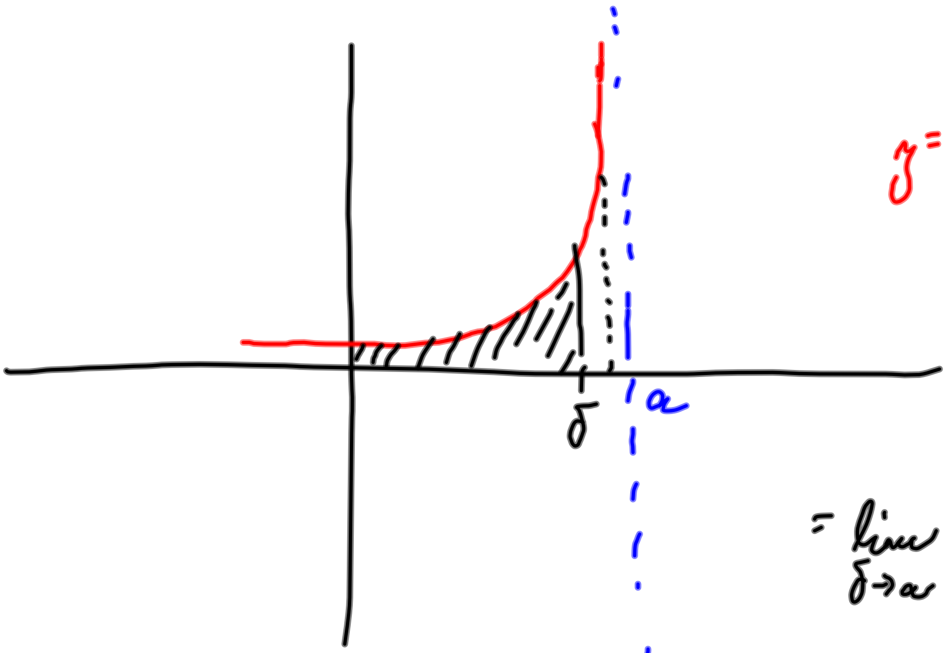


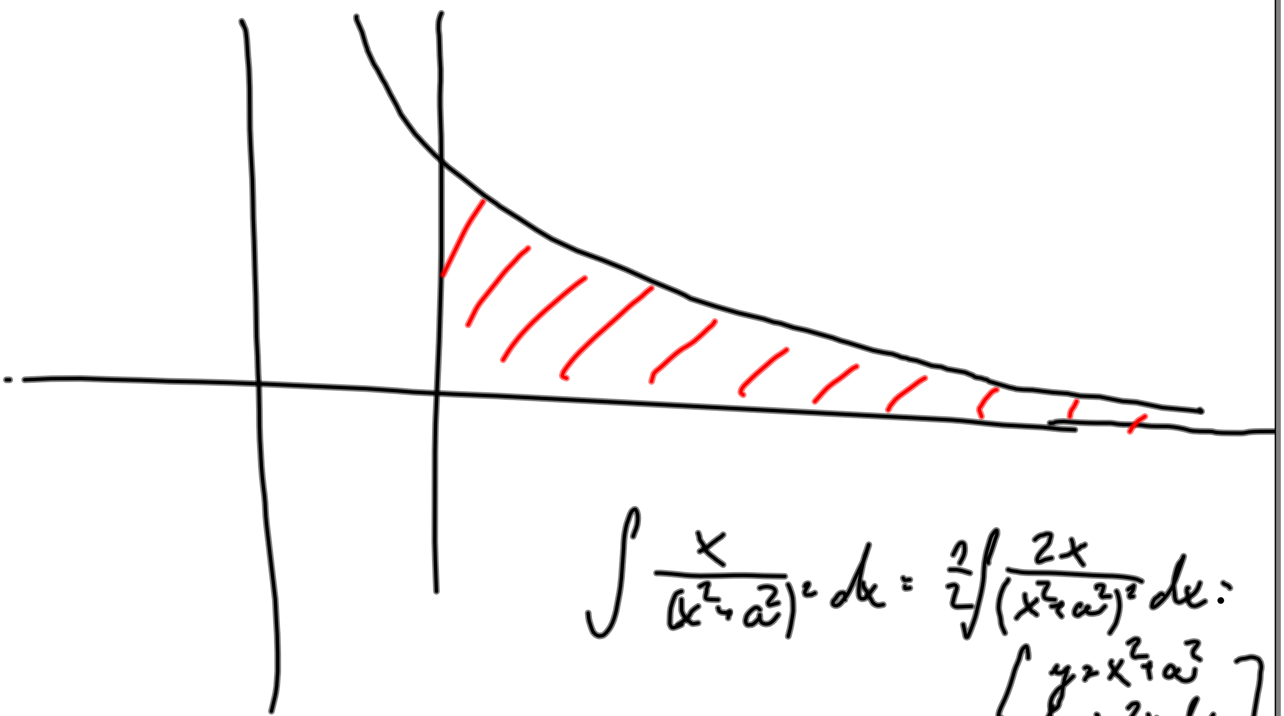
$$y = \frac{1}{(x-a)^2}$$

$$\int_0^a \frac{1}{(x-a)^2}\,dx$$

$$= \lim_{\delta \to a} \int_0^{\delta} \frac{1}{(x-a)^2}\,dx$$

$$\int_0^{2-\delta} \frac{1}{\sqrt[5]{2-x}}\,dx = \int_2^{\delta} -y^{-\frac{1}{4}}\,dy = \int_{\delta}^{2} y^{-\frac{1}{4}}\,dy$$

$$\begin{bmatrix} y = 2-x \\ dy = -dx \end{bmatrix}$$

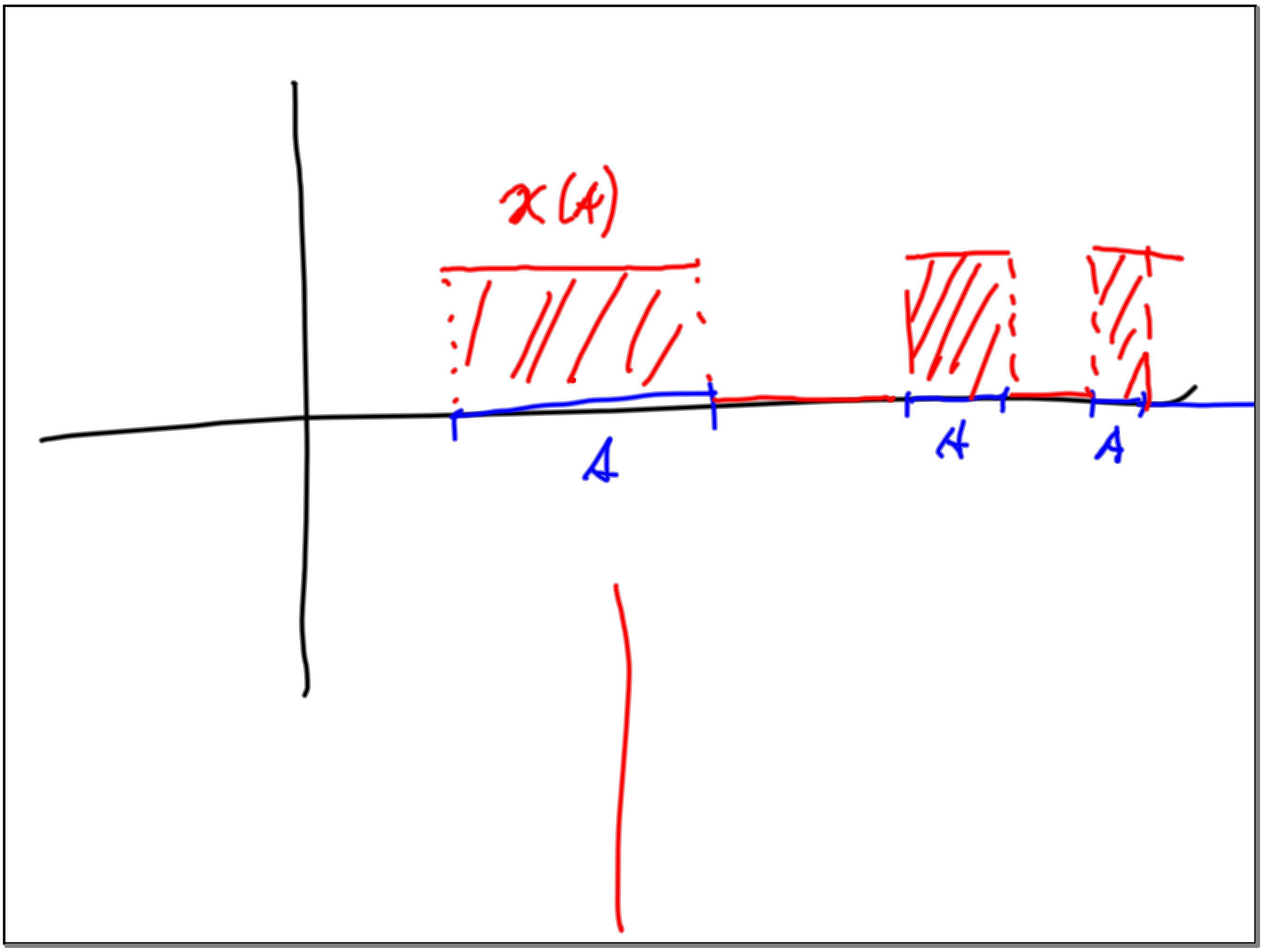


$$\int \frac{x}{(x^2+a^2)^2}\,dx = \frac{1}{2}\int \frac{2x}{(x^2+a^2)^2}\,dx =$$

$$\left[\begin{array}{l} y = x^2+a^2 \\ dy = 2x\,dx \end{array}\right]$$

$$= \frac{1}{2}\int \frac{1}{y^2}\,dy = -\frac{1}{2}\,\frac{1}{y} = -\frac{1}{2}\,\frac{1}{x^2+a^2}$$

$$= \frac{1}{2a^2}$$



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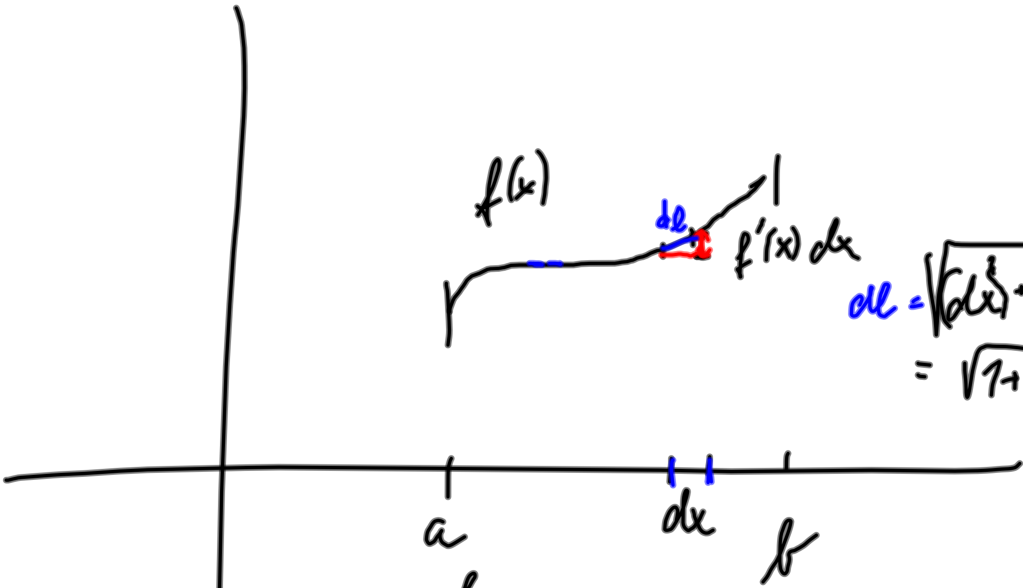
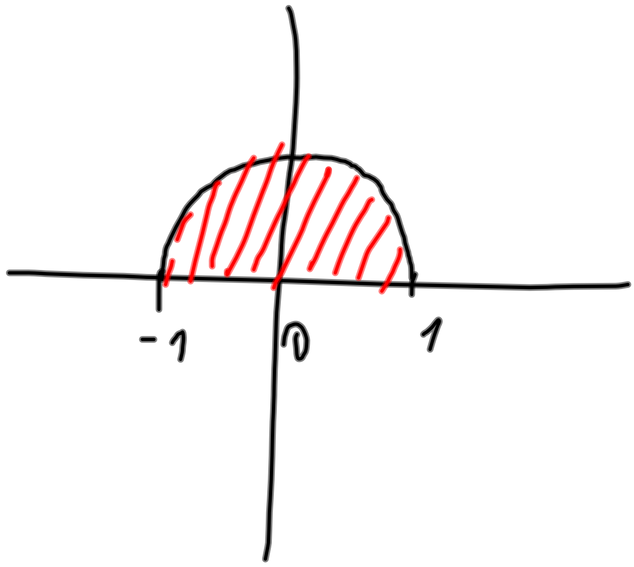
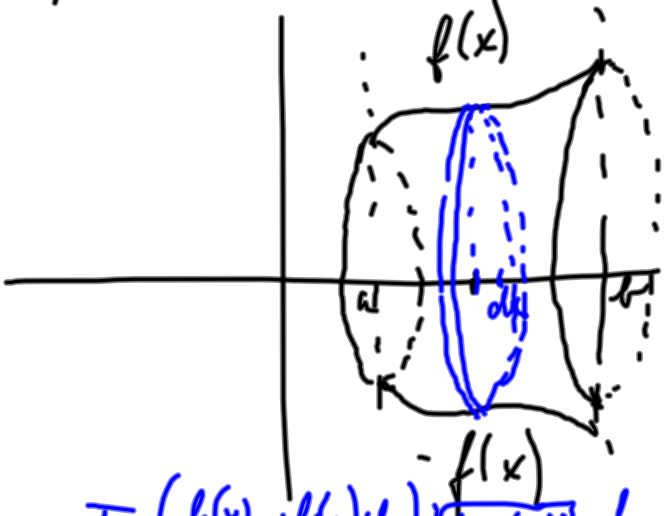


$$dl = \sqrt{(dx)^2 + f'(x)^2 (dx)^2}$$
$$= \sqrt{1+(f'(x))^2}\, dx$$

$$l = \int_a^b \sqrt{1+(f'(x))^2}\, dx$$



$$\pi\,(r_1+r_2)\sqrt{h^2+(r_2-r_1)^2}$$



$$\pi\left(f(x)+f(x+dx)\right)\sqrt{1+f'(x)^2}\,dx$$