subst $\quad tg\, x = t$

$$\frac{1}{\cos^2 x}\, dx = dt$$

$$I = \int \frac{\cos^4 x - \sin^2 x}{\cos^2 x}\, \cancel{\cos^2 x}\, dt$$

$$= \int \left(\cos^2 x - \frac{\sin^2 x}{\cos^2 x} \right) dt$$

$$t^2 = \frac{\sin^2}{\cos^2} = \frac{1 - \cos^2}{\cos^2}$$

$$= \int \left(\frac{1}{1+t^2} - t^2 \right) dt = \qquad \Rightarrow \cos^2 x = \frac{1}{1+t^2}$$

$$= \operatorname{arctg} t - \frac{t^3}{3}$$

$$I = \arctan(\tan x) - \frac{\tan^3 x}{3}$$

$$x - \frac{\tan^3 x}{3}$$

$$I' = 1 - \tan^2 x \, \frac{1}{\cos^2 x} = 1 - \frac{\sin^2 x}{\cos^4 x}$$

subst $\qquad t = x^3$

$$dt = 3x^2\,dx$$

$$I = \int \cancel{x^2}\sqrt{1-x^6}\,\frac{1}{3\cancel{x^2}}\,dt$$

$$= \frac{1}{3}\int \sqrt{1-t^2}\,dt \rightsquigarrow \text{Perpartes}$$

$$g(t) = t,\quad f(t) = \sqrt{1-t^2}$$

$$\frac{1}{3}\left(t\sqrt{1-t^2} - \int t\,\frac{1}{2}\,\frac{1}{\sqrt{1-t^2}}\cdot(-2t)\,dt \right.$$

$$I = \frac{1}{3} t\sqrt{1-t^2} + \frac{1}{3}\int \frac{t^2}{\sqrt{1-t^2}}\,dt$$

$$\int \underbrace{-\frac{(1-t^2)}{\sqrt{1-t^2}}}_{-I} + \frac{1}{\sqrt{1-t^2}}$$

$$2I = \frac{1}{3} t\sqrt{1-t^2} + \frac{1}{3}\arcsin t$$

$$I = \frac{1}{6}x^3\sqrt{1-x^6} + \frac{1}{6}\arcsin x^3$$

subst $t = \sqrt{1-x}$

$$I = \int \frac{\ln \frac{1}{t}}{t} (-2t)\, dt \qquad \begin{array}{l} x = 1 - t^2 \\ dx = -2t\,dt \end{array}$$

$$= -2 \int \ln \frac{1}{t}\, dt \quad \rightsquigarrow \text{per partes} \quad t,\ \ln\frac{1}{t}$$

$$= -2 \left(t \ln\frac{1}{t} - \int t \cdot \frac{1}{\frac{1}{t}} \cdot \frac{-1}{t^2} \right) dt$$

$$I = -2\left(t \ln \frac{1}{t} + t\right)$$

$$-2\sqrt{1-x}\left(\ln \frac{1}{\sqrt{1-x}} + 1\right)$$

Zk: $I' = \ldots$

subst $t = x^3$

$dt = 3x^2 dx$

$$I = \int \cancel{x^2} \ln x^6 \frac{1}{3\cancel{x^2}} \, dt$$

$$= \frac{1}{3} \int \ln t^2 \, dt \rightsquigarrow \text{per partes}$$

$t, \ln t^2$

$$= \frac{1}{3} \left(t \ln t^2 - \int t \frac{1}{t^2} \cdot 2t \, dt \right) =$$

$$\frac{1}{3}\left(t \ln t^2 - 2t\right)$$

$$\rightsquigarrow \underline{\underline{\frac{x^3}{3}\left(\ln x^6 - 2\right)}}$$

2k: $x^2\left(\ln x^6 - 2\right)$

$$+ \frac{x^3}{3} \frac{1}{x^6} \cdot 6x^5 =$$

$$\leftarrow \quad x^2 \ln x^6 \cancel{- 2x^2 + 2x^2} \quad \checkmark$$

$$I = \int \frac{1}{\cos^3 x}\, dx$$

subst $t = \sin x$

$dt = \cos x \, dx$

$$I = \int \frac{1}{\cos^3 x} \frac{1}{\cos x}\, dt$$

$$= \int \frac{1}{(1-\sin^2 x)^2}\, dt = \int \frac{1}{(1-t^2)^2}\, dt$$

$$(1-t^2)^2 = (1+t)^2(1-t)^2$$

$$\frac{1}{(1-t^2)^2} = \frac{A}{(1+t)^2} + \frac{B}{1+t} + \frac{C}{(1-t)^2} + \frac{D}{1-t}$$

$$1 = A(1-t)^2 + B(1-t)^2(1+t) + C(1+t)^2 + D(1+t)^2(1-t)$$

$t^3: \quad B - D = 0$

$$A(1-t)^2 + B(1-t^2)(1-t)$$

$$+\, C(1+t)^2 + D(1-t^2)(1+t)$$

t^3: $\quad 0 = B - D \quad \rightsquigarrow B = D$

t^2: $\quad 0 = A - B + C - D \quad \rightsquigarrow A = B$

t: $\quad 0 = -2A - B + 2C + D \quad \rightsquigarrow A = C$

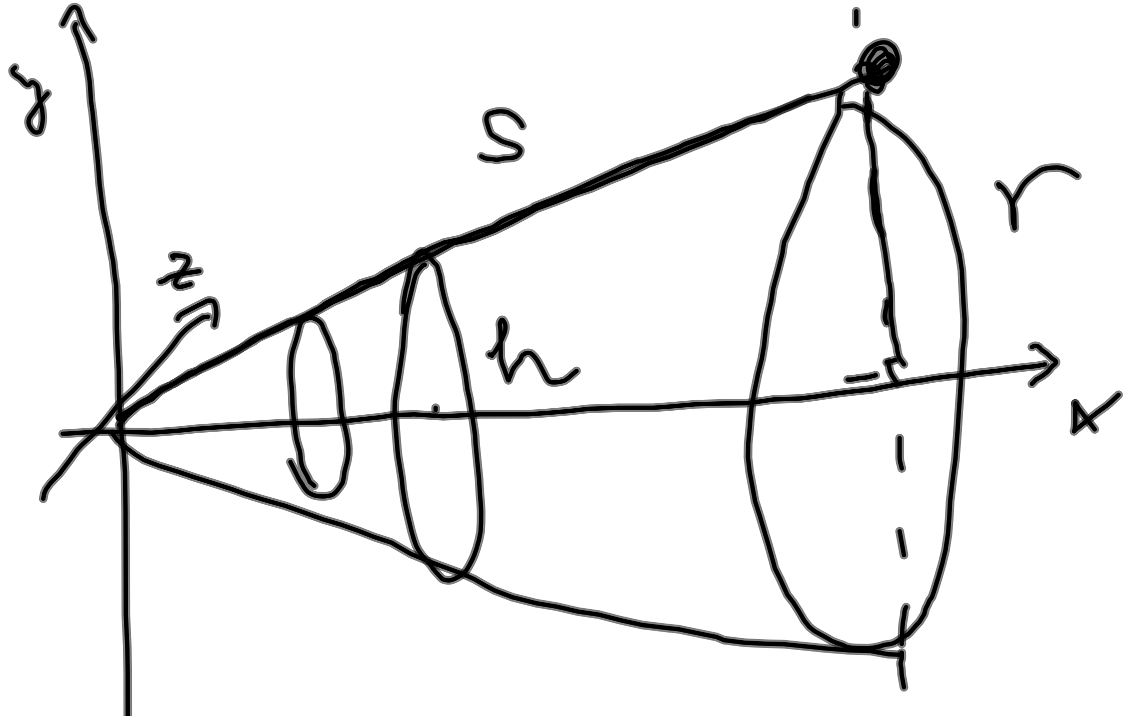
t^0: $\quad 1 = \underbrace{A + B + C + D}$

$$\rightsquigarrow A = B = C = D = \frac{1}{4}$$

$$\frac{1}{(1-t^2)^2} = \frac{1}{4}\left(\frac{1}{(1+t)^2} + \frac{1}{1+t} + \frac{1}{(1-t)^2} + \frac{1}{1-t}\right)$$

$$I = \frac{1}{4}\left(-\frac{1}{1+t} + \ln(1+t) + \frac{1}{1-t} - \ln(1-t)\right)$$

$$\rightsquigarrow \quad I = \frac{\sin x}{2\cos^2 x} + \frac{1}{4}\ln\frac{1+\sin x}{1-\sin x}$$



$$P_{pl} = 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$

$$\boxed{f(x) = \underline{k} \cdot x = \frac{r}{h}x}$$

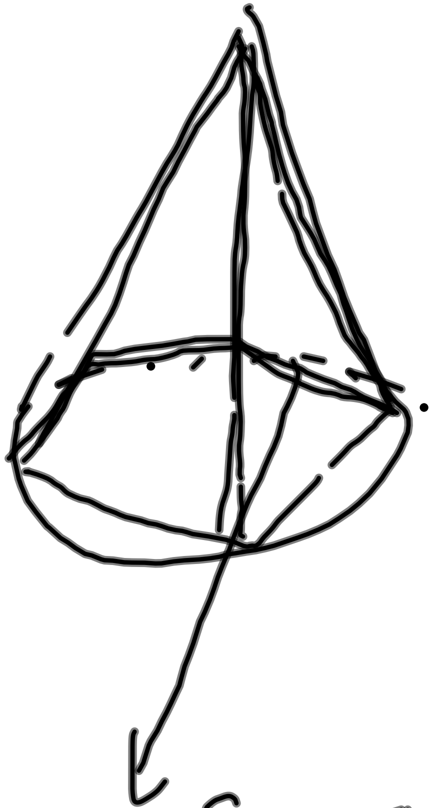
$$r = k \cdot h$$
$$k = \frac{r}{h}$$

$$P_{pl} = 2\pi \int_0^h \frac{r}{h} x \sqrt{1 + \left(\frac{r}{h}\right)^2}\, dx$$

$$= 2\pi \frac{r}{h} \sqrt{1 + \frac{r^2}{h^2}} \left[\frac{x^2}{2}\right]_0^h =$$

$$= \pi r h \sqrt{1 + \frac{r^2}{h^2}} = \pi r \sqrt{h^2 + r^2}$$

$$\left.\begin{aligned} &= \pi r s \\ P_{po} &= \pi r^2 \end{aligned}\right\} \Rightarrow P = \pi r s + \pi r^2$$

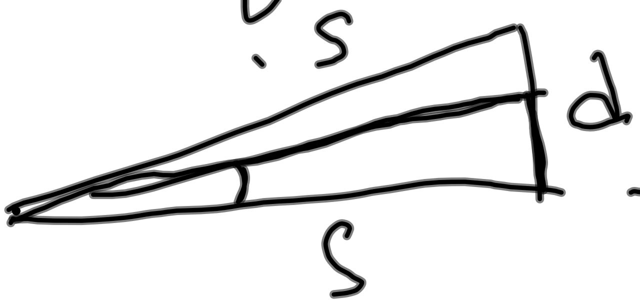


$$\varphi = \frac{2\pi}{n}$$

$\frac{\varphi}{2}$

$$S_1 = \frac{1}{2} r^2 \sin\varphi$$

$$d = 2r \sin\frac{\varphi}{2}$$



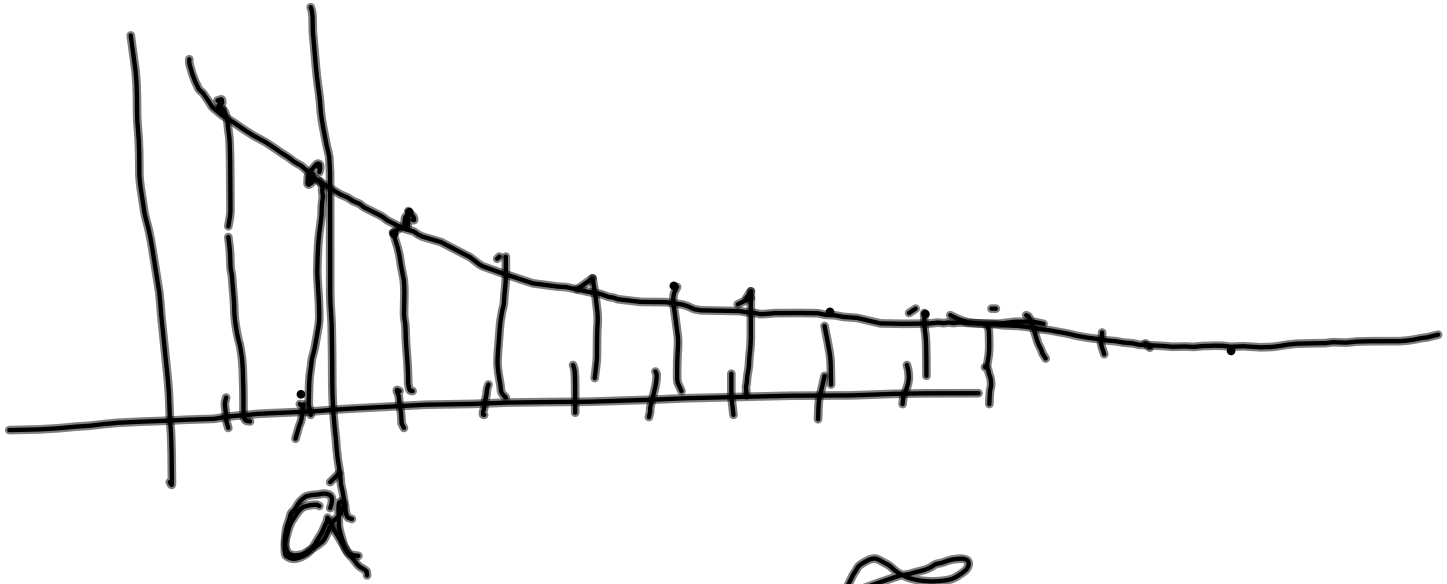
$$S_2 \quad s \cdot \cos\frac{\varphi}{2} \cdot \frac{d}{2}$$

$$\lim_{n\to\infty} : \qquad \boxed{P = \frac{n}{2} r^2 \sin\frac{2\pi}{n}}$$

$$V = \pi \int_a^b f^2(x)\,dx$$

$$= \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx = \frac{\pi r^2}{h^2}\int_0^h x^2$$

$$\frac{\pi r^2}{h^2}\left(\frac{x^3}{3}\right)_0^h = \underline{\frac{1}{3}\pi r^2 h}$$



$$\sum_{1}^{\infty} \frac{1}{n \ln n} \rightsquigarrow \int_{a}^{\infty} \frac{1}{x \ln x}\,dx$$

subst. $t = \ln x$ $\quad dt = \frac{1}{x}\,dx$

$$\int_{a}^{\infty} \frac{1}{t}\,dt = \ln t$$

$$\left[\ln\ln x\right]_A^{\infty} \longrightarrow \infty$$

$\Rightarrow$ řada diverguje

$x \to \infty$

$\ln x \to \infty$

$\ln\ln A \to \infty$

$$\int_{\ln 2}^{\ln 3} \underbrace{\sum_{1}^{\infty} n e^{-nx}}\ \stackrel{!}{=}$$

stejn. [illegible] sumabla

$$= \sum_{n=1}^{\infty} \int_{\ln 2}^{\ln 3} n e^{-nx}\, dx =$$

$$= \sum_{n=1}^{\infty} \left[-e^{-nx}\right]_{\ln 2}^{\ln 3} = \sum_{n=1}^{\infty} \left(e^{-n\ln 2} - e^{-n\ln 3}\right)$$

$$= \sum_{1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$$

$$\sum_{1}^{\infty}\left(\frac{1}{2^n}\right)-\sum_{1}^{\infty}\left(\frac{1}{3^n}\right)$$

$$\frac{\frac{1}{2}}{1-\frac{1}{2}}-\frac{\frac{1}{3}}{1-\frac{1}{3}}=1-\frac{1}{2}$$

$$=\underline{\underline{\frac{1}{2}}}$$

$$\sum_{1}^{\infty} \frac{1}{n 2^n} = \sum_{1}^{\infty} \int_{2}^{\infty} \frac{1}{x^{n+1}} \overset{!}{=}$$

$$= \int_{2}^{\infty} \sum_{1}^{\infty} \frac{1}{x^{n+1}} = \int_{2}^{\infty} \left(\frac{\frac{1}{x}}{1-\frac{1}{x}} - \frac{1}{x} \right) =$$

$$= \int_{2}^{\infty} \left(\frac{1}{x-1} - \frac{1}{x} \right) = \ln(x-1) - \ln x$$

$$= \left[\ln \frac{x-1}{x} \right]_{2}^{\infty}$$

$$\left[\ln \frac{x-1}{x}\right]_2^{\infty}$$

$$\ln\left(1-\frac{1}{x}\right) \quad x \to \infty \quad \ln 1 = 0$$

$$-\ln \frac{1}{2} = \underline{\underline{\ln 2}}$$