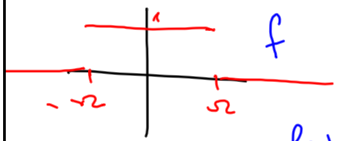
$f(x) \longmapsto \tilde{f}(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx$

$g_{odd}(x) = \frac{1}{2}(g(x) - g(-x))$

$g_{even}(x) = \frac{1}{2}(g(x) + g(-x))$

sinc



$\tilde{f} = \frac{2\sin \Omega\omega}{\omega}$

$\lim_{\omega \to 0} \tilde{f}(\omega)$

$\int_{-\infty}^{\infty} \frac{\sin t}{t}\, dt$

5 16-14:06

Diracova δ - funkce

$\delta(x) = \begin{cases} 0 & \text{pro } x \neq 0 \\ \infty & \text{pro } x = 0 \end{cases}$

$\int_{-\infty}^{\infty} g(x) \cdot \delta(x)\, dx = g(0)$

5 16-14:23

$\lim_{t \to 0} \frac{\sin \alpha t}{t} = \lim_{t \to 0} \frac{\alpha \cos \alpha t}{1} = \alpha$

$\int_{-\infty}^{\infty} \text{sinc}\, t\, dt$ konverguje?

ANO (srovnej s $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$)

5 16-14:30

Diskrétní verze: $r = 0, 1, \dots, N-1$

$f(r) \in \mathbb{C}$ konečný vektor

$\tilde{f}(k) = \frac{1}{N} \sum_{r=0}^{N-1} f(r)\, e^{-i\frac{2\pi}{N}kr}$

$f(k) \overset{?}{=} \sum_{r=0}^{N-1} \tilde{f}(r)\, e^{i\frac{2\pi}{N}kr}$

$k = 0, \dots, N-1$

$\Delta\omega \approx \frac{2\pi}{N}$

$\omega_k = k\Delta\omega$

Th. Skutečně platí.

5 16-14:46

Dk $\sum_{r=0}^{N-1} \frac{1}{N} \left(\sum_{s=0}^{N-1} x(s)\, e^{-i\frac{2\pi}{N}rs} \right) e^{i\frac{2\pi}{N}rk} = y(k)$

$= \frac{1}{N} \sum_{r=0}^{N-1} \sum_{s=0}^{N-1} x(s)\, e^{-i\frac{2\pi}{N}rs} \cdot e^{i\frac{2\pi}{N}rk}$

$= \frac{1}{N} \sum_{r=0}^{N-1} \sum_{s=0}^{N-1} x(s)\, e^{i\frac{2\pi}{N}r(k-s)}$

$= \frac{1}{N} \sum_{s=0}^{N-1} x(s) \sum_{r=0}^{N-1} e^{i\frac{2\pi}{N}r(k-s)}$

0 pro $k \neq s$

N pro $k = s$

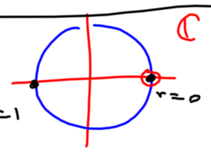
$= x(k)$

5 16-14:54

$\sum_{r=0}^{N-1} e^{ir\left(\frac{2\pi}{N}k\right)} = \begin{cases} N & \text{pro } N \text{ násobek } k \; ✓ \\ 0 & \text{jinak} \end{cases}$

$N = 2$

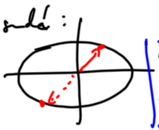
$k = 1$



obecné "počítání" v "$\mathbb{Z}_N$"

pro obecné N, a pevné k se rozpadne na "gtg".

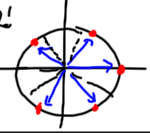
1) délka sudá:



$S_1 = A \cdot S_2$

$S_1 = -S_2$

2) délka lichá!



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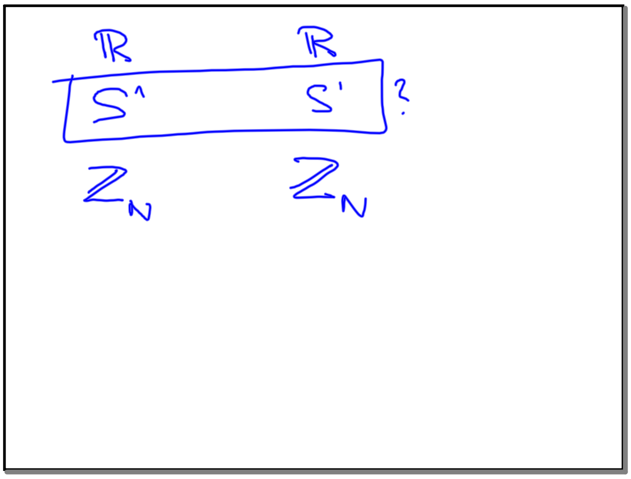

5 16-15:14

jiné vyjádření pro δ jako limita:

1) funkce impulz: $\delta_\alpha(x) = \begin{cases} \frac{1}{\alpha} & |x| \le \frac{\alpha}{2} \\ 0 & \text{jinde} \end{cases}$

2) "Gaussovský" $\delta_\alpha(x) = \frac{1}{\alpha\sqrt{\pi}} e^{-x^2/\alpha^2}$

3) "Lorentzův profil": $\delta_\alpha(x) = \frac{1}{\pi}\frac{\alpha}{x^2+\alpha^2}$

4) sinc: $\delta_\alpha(x) = \frac{1}{\alpha}\,\mathrm{sinc}\,\frac{\pi}{\alpha}x = \frac{\sin\frac{\pi}{\alpha}x}{\pi x}$

5 16-15:30