$f : \mathbb{R} \to \mathbb{R}, \mathbb{C}$ def. on A, $x_0 \in [a,b] = A$

$$f'(x_0) = \lim_{x \to x_0} c \cdot \frac{f(x) - f(x_0)}{x - x_0}$$

① $$f(x) = \frac{f(x) - f(x_0)}{x - x_0}(x - x_0) + f(x_0)$$

? $\downarrow_{x \to x_0}$ $\quad \boxed{f(x_0)} \quad f'(x_0) \cdot 0 \quad \boxed{f(x_0)}$

$$(f+g)'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0) + g(x) - g(x_0)}{x - x_0}$$

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$$\mathbb{R} \xrightarrow{f} \mathbb{R} \xrightarrow{h} \mathbb{R}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

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$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left[f(x + \Delta x) \cdot \frac{g(x + \Delta x) - g(x)}{\Delta x} + \frac{f(x + \Delta x) - f(x)}{\Delta x} \cdot g(x) \right]$$

$$= f(x) \cdot g'(x) + f'(x) \cdot g(x) \quad \checkmark$$

$$\left(\frac{f}{g}\right)' = \left[(g)^{-1} \cdot f\right]' = (g^{-1})' \cdot f + g^{-1} \cdot f'$$

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$$\mathbb{R} \underset{f^{-1}}{\overset{f}{\rightleftarrows}} \mathbb{R}$$

$f^{-1} \circ f = id_{\mathbb{R}}$

$f \circ f^{-1} = id_{\mathbb{R}}$

$$f' \doteq \frac{\Delta y}{\Delta x} \overset{?}{\Rightarrow} (f^{-1})' \doteq \frac{\Delta x}{\Delta y} = \frac{1}{f'}$$

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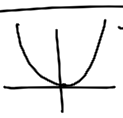
$$(a_0 x^n + \dots + a_n x^0)' = n a_0 x^{n-1} + \dots a_{n-1}$$

$$f(x) = (x - a_1) g(x) = \dots = (x - a_1)^{c_1} \dots$$

$$((f \cdot g) \cdot h)' = (f \cdot g)' \cdot h + (f \cdot g) \cdot h'$$
$$= f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$$



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$$(x^n)' = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{n \cancel{\Delta x} x^{n-1} + \binom{n}{2} (\Delta x)^2 x^{n-2} + \dots (\Delta x)^n}{\cancel{\Delta x}}$$

$$= n x^{n-1}$$

$y' = q x^{q-1}$

$x' = \frac{1}{q} \frac{1}{x^{q-1}}$

$$\frac{-q+1}{q} = -1 + \frac{1}{q}$$

$$(1/q)(p-1) + 1/q - 1$$

$$p/q - \cancel{1/q} + \cancel{1/q} - 1$$

$$(x^a)' = a \cdot x^{a-1}$$

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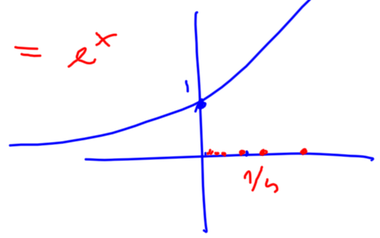
$$(a^x)' = ?$$

$$\boxed{x^a = e^{(\ln x)\cdot a}}$$

$$a^{x+\Delta x} = a^x \cdot a^{\Delta x}$$

$$(e^x)' = e^x$$



3 14-15:18

$$\frac{a^{1/n}-1}{1/n} \approx 1$$

$$a^{1/n} \approx 1 + 1/n$$

$$a \approx (1+1/n)^n$$

$$\boxed{\text{Ex. } \lim_{n\to\infty}(1+1/n)^n =: e}$$

$$\Rightarrow \exists \lim_{n\to\infty} \frac{a^{1/n}-1}{1/n} = 1$$

3 14-15:28

$$\frac{\left(1+\frac{1}{n}\right)^n}{\left(1+\frac{1}{n-1}\right)^{n-1}} = \frac{\left(\frac{n+1}{n}\right)^n}{\left(\frac{n}{n-1}\right)^{n-1}} = \frac{(n^2-1)^n\, n}{n^{2n}(n-1)}$$

$$e^{\boxed{\ln x + \ln y}} = e^{\ln x}\cdot e^{\ln y} = x\cdot y$$

$$\ln x + \ln y = \ln(x\cdot y)$$

$$\boxed{(a^x)'(0) = \ln a}$$

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$$(x^a)' = \dots = e^{a\ln x}\, a\cdot\frac{1}{x}$$

$$= a\cdot x^a\cdot\frac{1}{x} = a\, x^{a-1}$$

$$(x^x)' = (e^{x\ln x})' = e^{x\ln x}(x\ln x)'$$

$$= x^x\cdot(\ln x + 1)$$

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