$(a^x)' = \ln a \cdot a^x$ $\boxed{e^{\ln y} = y}$

$\lim_{x\to 0} \frac{a^x - 1}{x}$ $a^x \approx 1 + x$ $x = 1/n$

$a^{1/n} \approx 1 + \frac{1}{n}$ $a \approx \left(1+\frac{1}{n}\right)^n$

$\boxed{\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e}$ $\boxed{e^x = \lim_{n\to\infty} \underbrace{\left(1+\frac{x}{n}\right)^n}_{u_n}}$

? $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{2^k} = \lim_{n\to\infty} \frac{1-q^n}{1-q} \to \frac{1}{1-q}$ $|q|<1$

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$\boxed{e^x = \lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{k!} x^k =: \boxed{\sum_{k=0}^{\infty} \frac{x^k}{k!}}}$

derivace v 0 $\sum \frac{x^{k-1}}{k!}$

$\lim_{x\to 0_+} \frac{e^x - 1}{x} = \lim_{x\to 0_+} \left(\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k!} x^{k-1} \right)$

$= \lim_{x\to 0_+} \underbrace{\left(1 + \frac{1}{2}x + \dots\right)}_{< e^x}$

$= 1$

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$\boxed{f' = f}$

$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$

$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

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$c_n = \sum_{k=0}^{n} a_{n-k} \cdot b_k$ $\lim_{n\to\infty} c_n$?

$\left(\sum_{n=0}^{k} a_n\right) \cdot \left(\sum_{n=0}^{k} b_n\right) \longrightarrow \left(\sum_{n=0}^{\infty} a_n\right) \cdot \left(\sum_{n=0}^{\infty} b_n\right)$

$\downarrow$ $\sum_{n=0}^{\infty} a_n$ $\downarrow$ $\sum_{n=0}^{\infty} b_n$

$\left[\left(\sum_{n=0}^{k} a_n\right) \cdot \left(\sum_{n=0}^{k} b_n\right) - \sum_{n=0}^{k} c_n\right] \overset{?}{\to} 0$

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$\lim_{n\to\infty} a_n \not\to 0 \Rightarrow \exists\, \varepsilon > 0 \dots$

$\Rightarrow$ nekonečně mnoho $|a_k| > \varepsilon$

$\Rightarrow$ nekonečně mnoho kladných nebo záp.

$\Rightarrow$ změní se vždy [illegible] $S_{n+1} \to S_n$ o aspoň ε

$\Rightarrow$ není Cauchyovská

$\Rightarrow$ není konvergentní.

ε 0 $\mathbb{R}$

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2) předp. $a_i \in \mathbb{R}$, $a_i > 0$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = q < 1 \Rightarrow$ pro velká $j > N$

$a_{j+1} < q \cdot a_j < q^{-(j-N+1)} c_N$

$S_n < \sum_{j=0}^{N} a_j + c_N \underbrace{\sum_{j=0}^{n-N} \frac{1}{q^j}}$ $\to \frac{1}{1-q}$

3) $\sqrt[n]{a_n} < r$ $\boxed{q < r < 1}$

$a_n < r^n$ $\boxed{1 < r < q}$ $a_n > r^n$

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$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots \qquad \rho = \infty$

$a_n = \frac{1}{n!} \qquad \boxed{\sqrt[n]{a_n} = \sqrt[n]{\frac{1}{n!}} \xrightarrow{?} 0}$

[illegible]:

$$\lim_{n\to\infty} \sqrt[n]{a_n x^n} = \Big(\underbrace{\lim_{n\to\infty} \sqrt[n]{a_n}}_{=\rho}\Big)\, x$$

< 1 konv.

> 1 div.

3 21-15:10

$$\left|\frac{a_{n+1}x^{n+1}}{a_n x^n}\right| = \left|\frac{a_{n+1}}{a_n}\right| \cdot x$$

$$\underbrace{1 + \frac{1}{2}}_{>1/2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{>1/2} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{>4\cdot\frac{1}{8} = \frac{1}{2}} + \dots$$

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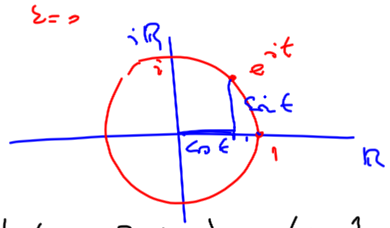
$$\left(\sum_{k=0}^{\infty} \frac{1}{k!} x^k\right) \cdot \left(\sum_{k=0}^{\infty} \frac{1}{k!} y^k\right) = \sum_{k=0}^{\infty} \frac{1}{k!}(x+y)^k \qquad \checkmark$$

$|e^{it}| = 1$



$$e^{it} = \underbrace{\left(1 - \frac{1}{2}t^2 + \frac{1}{4}t^4\right)}_{\cos t} + i\underbrace{\left(t - \frac{1}{3!}t^3 + \dots\right)}_{\sin t}$$

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