HOMOMORFISMUS:
mějme grupy $(G,\cdot),(H,\square)$
Zobrazení $f: G \to H$ nazýváme homomorfismus, jestliže $\forall a,b \in G$ platí, že
$f(a\cdot b) = f(a) \square f(b)$
$\in H \quad \in H$

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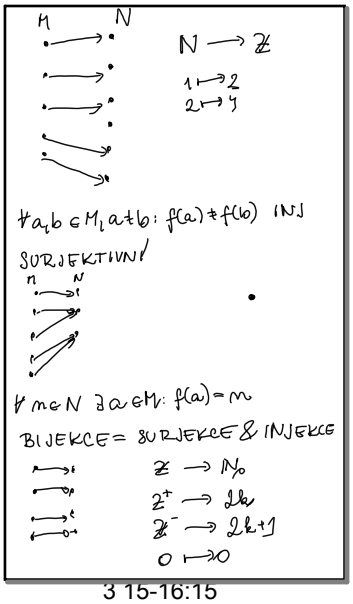
PŘ.: $\varphi(\mathbb{Z},+) \to (\mathbb{Z},+)$
$\varphi(a) = 2a$
Homomorfismus? ANO!
$a,b \in \mathbb{Z}$
$f(a+b) = 2(a+b)$
$f(a)+f(b) = 2a+2b = 2(a+b)$ =

3 15-16:05

PŘ.: $(G,\cdot)$ ANO
$G = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \middle| a,b,c \in \mathbb{Z} \right\}$
$f: (G,\cdot) \to (\mathbb{Z},+) \quad f(A) = a - c$
$A,B \in G$
$f(A\cdot B) = f\left(\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix}\right) =$
$= f\left(\begin{pmatrix} 1 & a+d & e+af+b \\ 0 & 1 & f+c \\ 0 & 0 & 1 \end{pmatrix}\right) = a+d-f-c$
$f(A)+f(B) = f\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} + f\begin{pmatrix} 1 & d & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix} =$
$= a-c+d-f = a+d-f-c$ =

3 15-16:08



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JÁDRO: $f: G \to H$ hom
$\ker f = \{ g \in G \mid f(g) = e_H \} \in G$
OBRAZ: $f: G \to H$
$\mathrm{Im} f = \{ h \in H \mid \exists g \in G: f(g) = h \}$
$\in H$
VĚTA: Hom f je INJ $\iff \ker f = \{e_G\}$
DEF: Izomorfismus je bijektivní homomorfismus.

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PŘ.: $\pi: (\mathbb{R}^2,+) \to (\mathbb{R},+)$
$\pi(x,y) = x$
hom? jádro? obraz?
$\pi((x,y)+(\bar{x},\bar{y})) = \pi(x+\bar{x}, y+\bar{y}) =$
$= x+\bar{x}$ =
$\pi(x,y)+\pi(\bar{x},\bar{y}) = x+\bar{x}$
Jádro: $\ker \pi = \{(x,y) \mid \pi(x,y) = 0\}$
$= \{(0,a) \mid a \in \mathbb{R}\}$
Obraz: $\mathrm{Im}\,\pi = \{x \in \mathbb{R} \mid \exists (u,v): \pi(u,v) = x\}$
$\mathrm{Im}\,\pi = (\mathbb{R},+)$
projekce na 1. složku.

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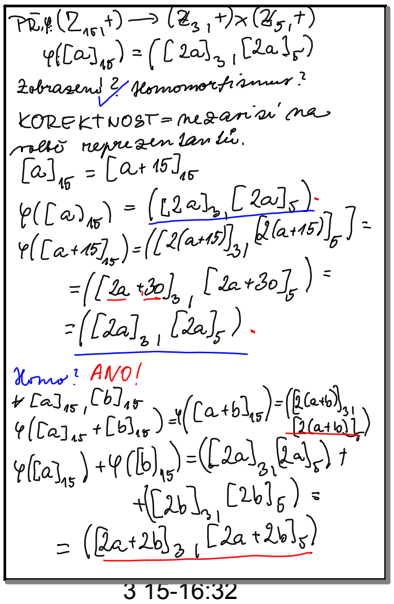

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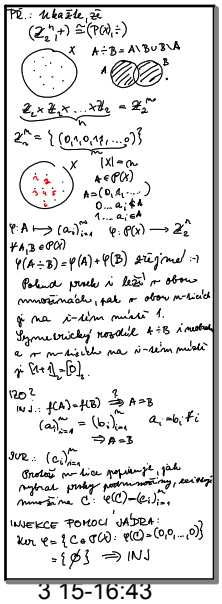

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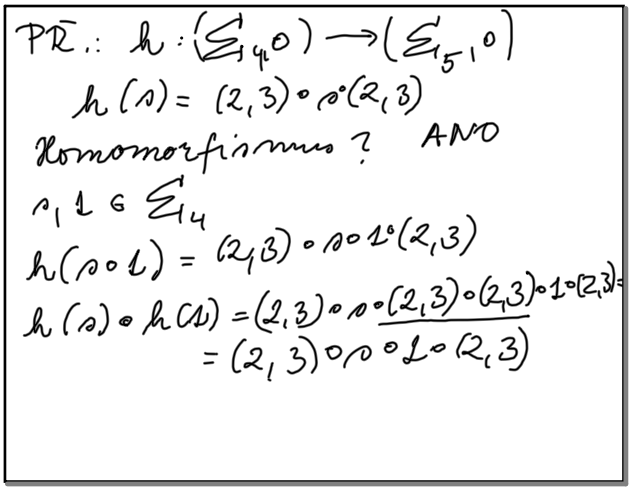

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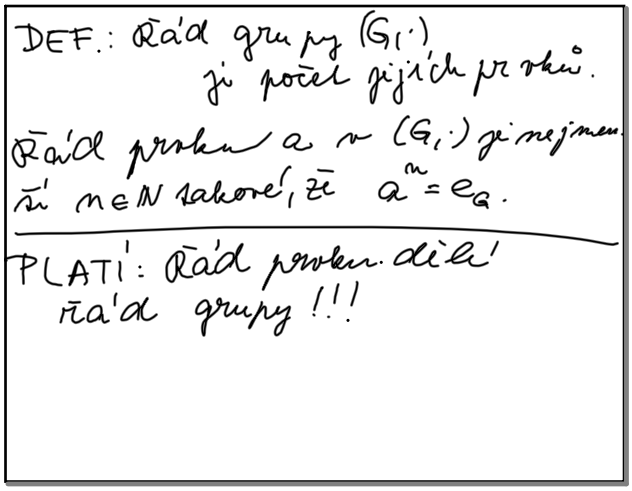

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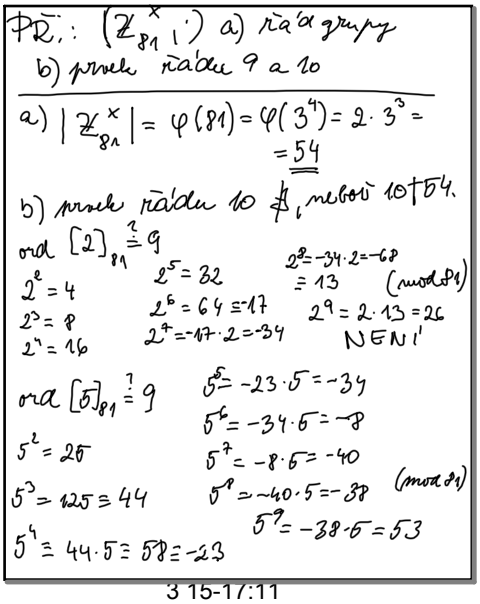

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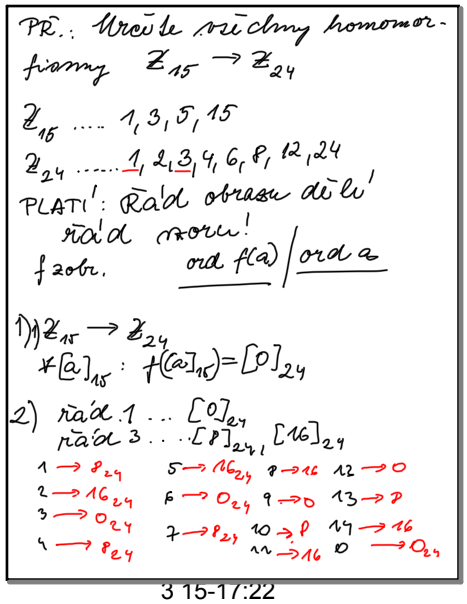

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