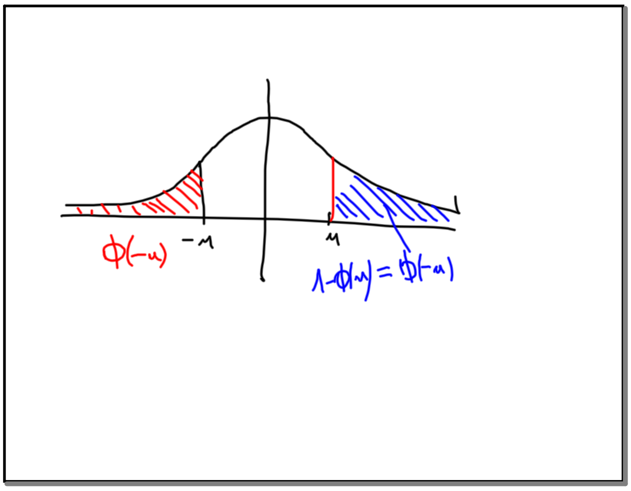
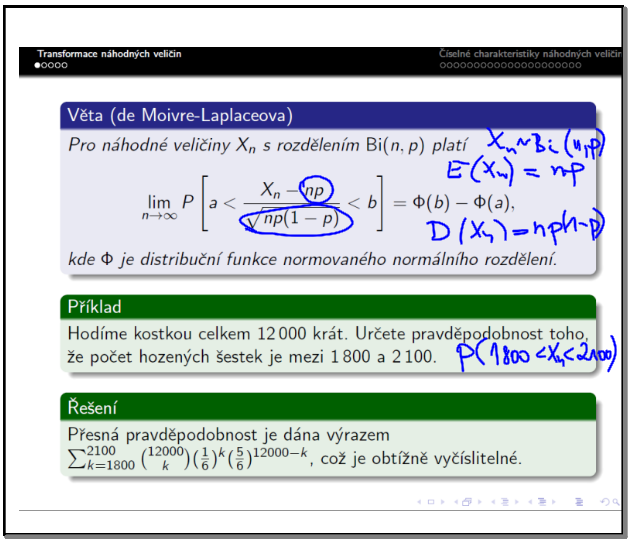
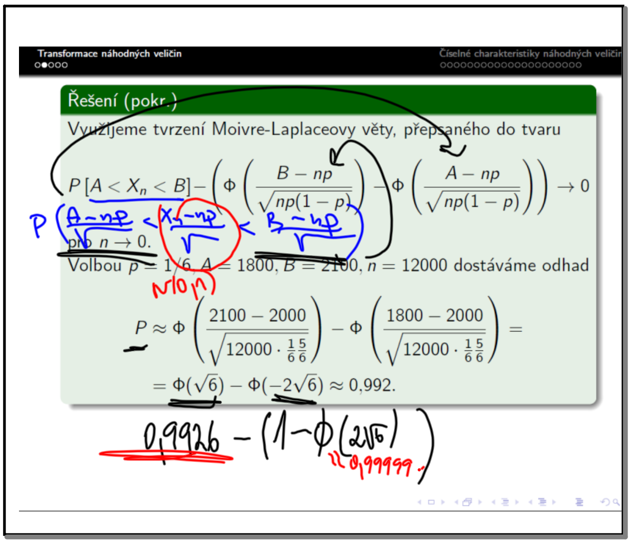


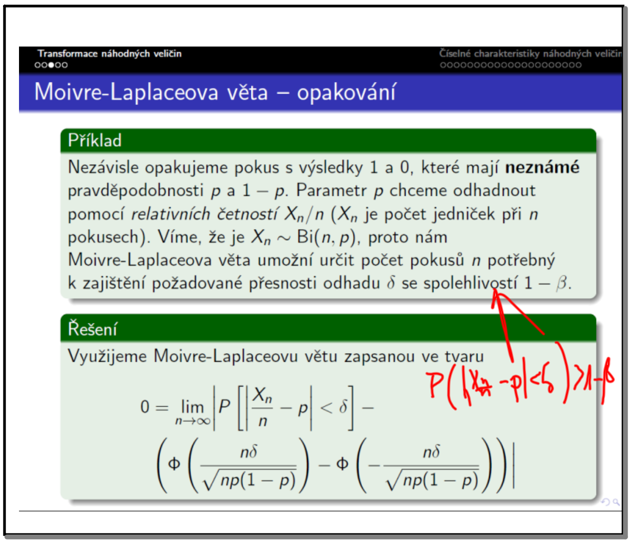
4 27-14:04



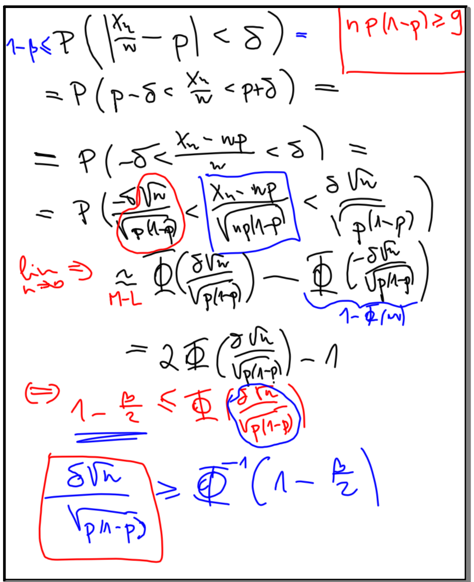
4 27-14:13



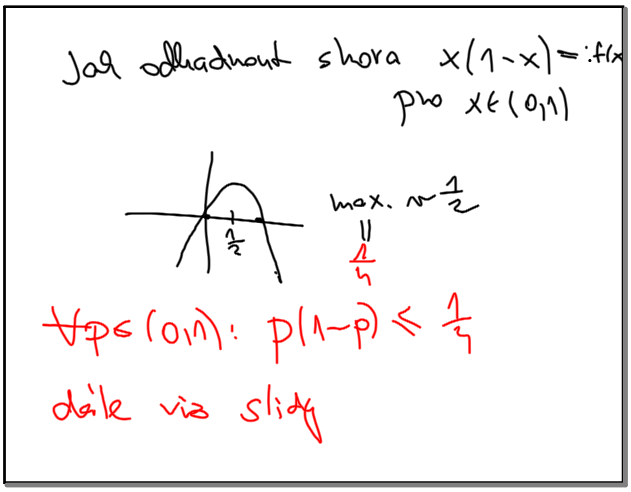
4 27-14:17



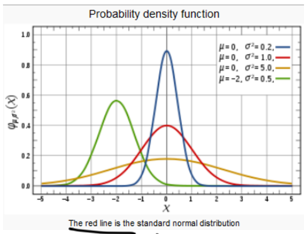
4 27-14:23



4 27-14:25



4 27-14:15



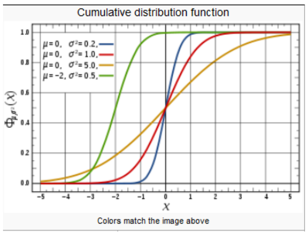
$N(\mu, \sigma^2)$

hustota

$N(0,1)$

stř. h. rozptyl

distribuční fce



4 27-14:08

Transformace náhodných veličin — Číselné charakteristiky náhodných veličin

Příklad

Spočtěme střední hodnotu binomického rozdělení.

Řešení

$f_X(k) = P(X=k)$

Pro $X \sim \mathrm{Bi}(n,p)$ je

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$E(X) = \sum_{k=0}^{n} k \cdot \binom{n}{k} p^k (1-p)^{n-k} =$$

$k = j+1$

$$= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-k)!(k-1)!} p^{k-1}(1-p)^{n-k} =$$

$$= np \sum_{j=0}^{n-1} \boxed{\frac{(n-1)!}{(n-1-j)!j!}} p^j (1-p)^{n-1-j} =$$

$= \binom{n-1}{j}$

$$= np(p + (1-p))^{n-1} = np.$$

$$(A+B)^{n-1} = \sum_{j=0}^{n-1} \binom{n-1}{j} \cdot A^j B^{n-1-j}$$

4 27-14:11

• $E(X+Y) = E(X) + E(Y)$

ve spojitém případě

$f_{X,Y}(x,y)$

$$E(X+Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y)\, P(X=x, Y=y)\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} x \cdot f_{X,Y}(x,y)\, dy\right) dx + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \cdot f_{X,Y}(x,y)\, dx\, dy$$

$$= \underbrace{\int_{-\infty}^{\infty} x \cdot f_X(x)\, dx}_{E(X)} + \underbrace{\int_{-\infty}^{\infty} y \cdot f_Y(y)\, dy}_{E(Y)}$$

4 27-14:58

• v diskrétním případě

$$E(X+Y) = \sum_{i,j} (x_i + y_j)\, P(X=x_i, Y=y_j)$$

$$= \sum_i \underbrace{\sum_j x_i \boxed{P(X=x_i, Y=y_j)}} + \sum_j \sum_i y_j P(X=x_i, Y=y_j)$$

$$= \underbrace{\sum_i x_i\, P(X=x_i)} + \sum_j y_j\, P(Y=y_j)$$

$$= E(X) + E(Y)$$

4 27-15:04

X, Y stochasticky nezávislé:

$P(X=x_i, Y=y_j) = P(X=x_i) \cdot P(Y=y_j)$

$F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y)$ distr.

$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$ hust.

$p_{X,Y}(x,y) = p_X(x) \cdot p_Y(y)$ pravd. fce

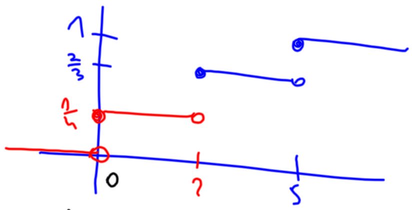
spojité: $E(X \cdot Y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x \cdot y\, f_{X,Y}(x,y)\, dx\, dy$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot y \cdot f_X(x) \cdot f_Y(y)\, dx\, dy =$$

$$= \int_{-\infty}^{\infty} y f_Y(y) \underbrace{\left(\int_{-\infty}^{\infty} x f_X(x)\, dx\right)}_{E(X)} dy =$$

$$= E(X) \cdot \int_{-\infty}^{\infty} y f_Y(y)\, dy = E(X) \cdot E(Y).$$

4 27-15:08

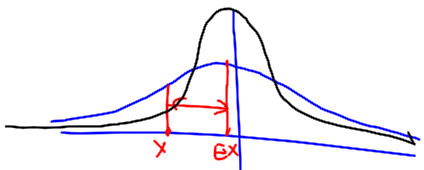


$F^{-1}(\alpha) = \inf\{x \in \mathbb{R};\ F(x) \geq \alpha\}$

$F^{-1}(0,1) = 0$

$F^{-1}\left(\frac{1}{4}\right) = 0 \quad F^{-1}\left(\frac{1}{3}\right) = 2, \ldots$

4 27-15:13



$$E\left(|X-EX|^2\right)$$

4 27-15:22

ad 1.

$$D(X) = E\left((X-EX)^2\right) =$$
$$= E\left((X-EX)\cdot(X-EX)\right) =$$
$$= E\left(X^2 - 2X\cdot EX + (EX)^2\right) =$$
$$= E(X^2) - E(2X\,EX) + E\left((EX)^2\right) =$$
$$= E(X^2) - 2EX\cdot E(X) + (EX)^2$$
$$= E(X^2) - (EX)^2$$

4 27-15:26

$$D(a+bX) = E\left((a+bX)^2\right) - \left(E(a+bX)\right)^2$$
$$= E\left(a^2 + 2abX + b^2X^2\right) -$$
$$- \left(a + b\cdot EX\right)^2 =$$
$$= \cancel{a^2} + \cancel{2ab}\cdot EX + b^2 E(X^2)$$
$$- \left(\cancel{a^2} + \cancel{2ab}EX + b^2(EX)^2\right) =$$
$$= b^2\left(E(X^2) - (EX)^2\right) = b^2 DX$$

4 27-15:28

normované náhodné veličiny

X ... EX, DX

$$\Rightarrow Y = \frac{X-EX}{\sqrt{DX}} \quad \Rightarrow \quad EY = 0, \quad DY = 1$$

$$\left(EY = E\left(\frac{X-EX}{\sqrt{DX}}\right) = \frac{1}{\sqrt{DX}}\cdot E(X-EX) =\right.$$
$$= \frac{1}{\sqrt{DX}}\cdot\left(E(X) - EX\right) = \underline{\underline{0}}$$

$$DY = D\left(\frac{X-EX}{\sqrt{DX}}\right) = \frac{1}{DX}\cdot D(X-EX)$$
$$= \frac{1}{DX}\cdot DX = \underline{\underline{1}}$$

4 27-15:35