

Classwork N°.2  
due to 2nd March 2012

1. Combinatory base : **S** and **K**

We have seen that the primitive combinators **S** and **K** can define all other combinators such as **C**, **I**, **B**, etc. Thus, it should be possible to prove it using these two primitive combinators. Show the completeness of the S-K basis by answering to the following question.

**Question:** Calculate the given combinatorial expressions using the definitions:

$$\mathbf{S}xyz := xz(yz)$$

$$\mathbf{K}xy := x$$

$$\mathbf{I}x := x$$

(1) **S K K x**

(2) **S (K S) K x y z**

(3) **S (K (S I)) K x y**

(4) **S (K (S I)) (S (K K) I) x y**

2. Abstraction algorithm

$T[]$  may be defined as follows:

1'|  $T[v] \Rightarrow v$

2'|  $T[(E1 E2)] \Rightarrow (T(E1) T(E2))$

3'|  $T[\lambda x.E] \Rightarrow (\mathbf{K} T[E])$

4'|  $T[\lambda x.x] \Rightarrow \mathbf{I}$

5'|  $T[\lambda x.\lambda y.E] \Rightarrow T[\lambda x.T[\lambda y.E]]$

6'|  $T[\lambda x.(E1 E2)] \Rightarrow (\mathbf{S} T[\lambda x.E1] T[\lambda x.E2])$

**Question:** Convert the following lambda terms into a combinator using the  $T[]$  definition:

(5)  $\lambda x.\lambda y.(yx)$

(6)  $\lambda x.\lambda y.(xy)$