Classwork N°3
due to 9th March 2012

1. Normal form and Interdefinability of simple combinators

When a combinatorial expression X cannot be anymore reduced by reaching to an expression x, we say that x is the normal form of X. Give the normal form of the given combinatorial expressions. The β-reduction rule of the basic combinators is given in the following:

$$\mathbf{B}xyz \geq_\beta x(yz)$$

$$\mathbf{C}xyz \geq_\beta x(zy)$$

$$\mathbf{S}xyz \geq_\beta xz(yz)$$

$$\mathbf{I}x \geq_\beta x$$

$$\mathbf{K}xy \geq_\beta x$$

$$\mathbf{W}xy \geq_\beta xyy$$

$$\mathbf{\Phi}xyzu \geq_\beta x(yu)(zu)$$

$$\psi\, xyzu \geq_\beta x(yz)(yu)$$

(1) $\mathbf{Wk}x \rightarrow kxx \rightarrow x$ **[I=WK]**

(2) $\mathbf{BCC}xyz \rightarrow C(Cx)yz \rightarrow Cxzy \rightarrow xyz$ **[BCC=I]**

(3) $\mathbf{CSI}fx \rightarrow SfIx \rightarrow fx(Ix) \rightarrow fxx$ **[W=S(CI)]**

(4) $\mathbf{SS(KI)}fx \rightarrow SfIx \rightarrow fx(Ix) \rightarrow fxx$ **[SS(KI)=W]**

(5) $\mathbf{B(BS)B}fxyz \rightarrow BS\,(Bf)xyz \rightarrow S(Bfx)\,yz \rightarrow Bfxz(yz) \rightarrow f(xz)(yz)$ **[B(BS)B=Φ]**

(6) $\mathbf{BB(BB)}fgxgy \rightarrow B(BBf)gxgy \rightarrow BBf(gx)(gy) \rightarrow B(f(gx))gy \rightarrow f(gx)(gy)$

(7) $\mathbf{S(BBS)(KK)}xyz \rightarrow BBSx(KK)xyz \rightarrow BBSxKyz \rightarrow B(Sx)Kyz$
$\rightarrow Sx(Ky)z \rightarrow xz(Kyz) \rightarrow xzy$ **[S(BBS)(KK)=C]**

(8) $\mathbf{B(BW(BC))(BB(BB))}fgxy \rightarrow B(BW(BC))Xfgxy \rightarrow BW(BC)(Xf)gxy \rightarrow W(BC(Xf))gxy \rightarrow$
$BC(Xf)ggxy \rightarrow C(Xfg)gxy \rightarrow Xfgxgy \rightarrow f(gx)(gy)$ **[B(BW(BC))(BB(BB))=ψ]**

(9) $\mathbf{\Phi(\Phi(\Phi B))B(KK)}fgxy \rightarrow \Phi(\Phi\ B)(Bf)(KKf)gxy \rightarrow \Phi\ (\Phi\ B)(Bf)Kgxy \rightarrow \Phi\ B(Bfg)(Kg)xy$

$\rightarrow B(Bfgx)(Kgx)y \rightarrow B(f(gx))gy \rightarrow f(gx)(gy)$ **[$\Phi(\Phi(\Phi B))B(KK)=\psi$]**

Please comment the definitions that you could find by reducing the given combinators. For example, is the definition **[W≡SS(KI)]** *an acceptable definition according your calculus?*