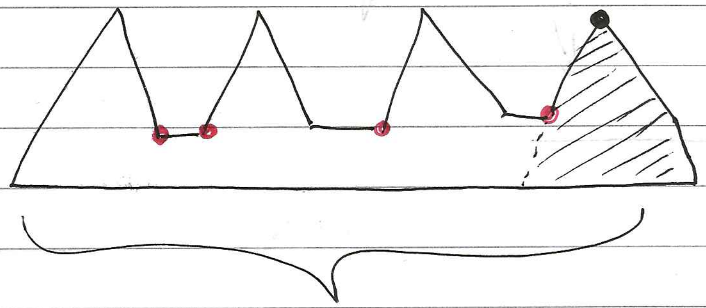
PROBLEM: ART GALLERY

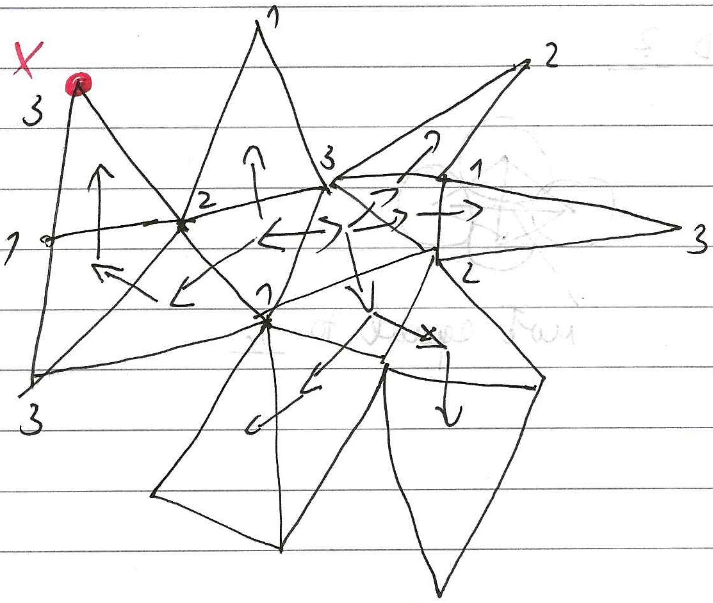
IN A (NON-CONVEX) N-GON, called P, PLACE THE LEAST NUMBER g OF GUARDS, SUCH THAT EVERY SIDE OF P IS WATCHED BY SOME GUARD.

THEOREM: FOR EVERY POSSIBLE N-GON P, $\lfloor M/3 \rfloor$ GUARDS SUFFICE.

ALGORITHMICALLY SOLVABLE ?

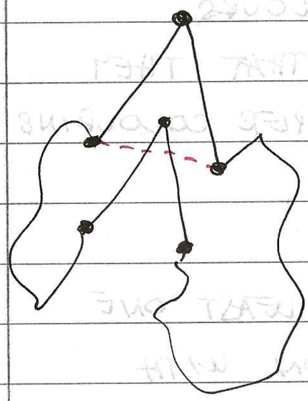


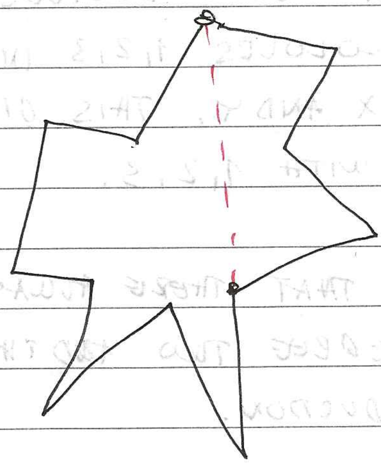
$k \to M = 3k$

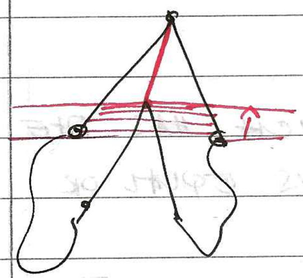


1. TRIANGULATION OF P
2. FIND VERTEX WITH NO DIAGONAL, X
3. TAKE P' = (P WITHOUT X) BY INDUCTION, ASSIGN "COLOUR" TO THE VERTEXES OF P' WITH 1, 2, 3
4. A COLOUR 1, 2 OR 3 IS FIT FO
5. TAKE AS GUARDS THE SMALLEST OF THE GROUPS #1, #2, #3.

THEOREM: THERE IS AN EFFICIENT ALGORITHM FOR FINDING THE PLACEMENT OF AT MOST $\lfloor M/3 \rfloor$ GUARDS.







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HOW TO FIND THE TRIANGULATION:

P HAS A CONVEX VERTEX (ANGLE $< 180°$), BECAUSE OF THE CONVEX HULL OF P. SAY IT IS X WITH TWO NEIGHBOURS U, V. IT IS TRIANGLE. XUV IS DISTINCT FROM THE REST OF P, THAN MAKE P' BY REMOVING X (REPLACE WITH A SIDE UV), AND BY INDUCTION, P' CAN BE TRIANGULATED.

IF XUV CONTAINS ANOTHER VERTEX OF P, THEN SLIDE THE LINE UV TOWARDS X UNTIL THE LAST LINE HITS A POINT W. THEN W IS A VERTEX OF P AND THE SEGMENT XW DOES NOT CROSS THE BOUNDARY OF P. NOW MAKE P_1, P_2 BY CUTTING P ALONG UW, AND TRIANGULATE P_1 AND P_2 SEPARATELY BY INDUCTION

COLOURING THE TRIANGULLATION T OF P BY 1,2,3:

TAKE ANY INTERIOR EDGE XY OF TRIANGULATION T, AND SPLIT T INTO T_1, T_2 ALONG IT. BY INDUCTION, T_1 AND T_2 CAN BE INDEPENTLY COLOURED WITH 1,2,3 SUCH THAT EVERY EDGE RECIVES TWO COLOURS. PERMUTE THE COLOURS 1,2,3 IN T_2 SO THAT THEY MATCH T_1 ON X AND Y. THIS GIVES A PROPER COLOURING OF $T = T_1 \cup T_2$ WITH 1,2,3.

(OTHER WAY) PROVE THAT THERE ALWAYS IS AT LEAST ONE VERTEX OF DEGREE TWO AND THEN GO ON WITH LINEAR INDUCTION.

PLACING THE GUARDS:

CHOOSE AMONG 1,2,3 THE NUMBER WHICH HAS THE LEAST NUMBER OF OCCURENCIES ON T. THIS IS EQUAL OR LESS THAN $\lfloor n/3 \rfloor$ GUARDS.

$\square$

HW: CONVEX POLYGONS IN 3D TOUCHING EACH OTHER BY A FACE?