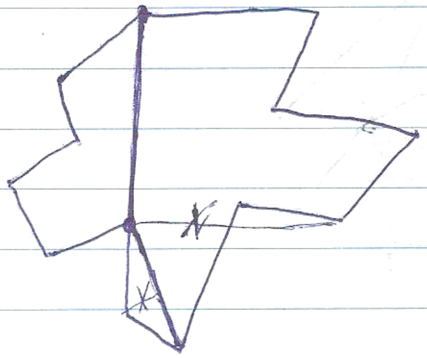
ART GALLERIES ~ EXTENSION

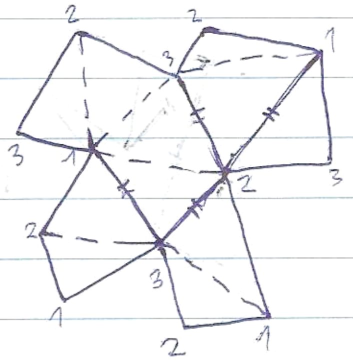
art galleries with interior walls (touching two vertices of ~~outer~~ the exterior wall)



possible approaches:
① "cloning guards"
② "obstructors"
③ "karma points"

CLONING GUARDS

① triangulation using interior walls, 3-colouring
- best approach for small number of interior walls

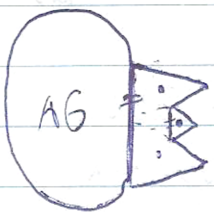


$$\sum_{v \in V} (d(v) - 1) = 2n + 2m - n \longrightarrow \underline{\underline{\left\lfloor \frac{n+2m}{3} \right\rfloor G}}$$

EVEN M

$m < \left\lfloor \frac{2n}{5} \right\rfloor$

$m = 2k$
$n - 5k$



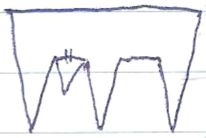
V +5
IW +2
G +3

$$G = \left\lfloor \frac{n-5k}{3} \right\rfloor + 3k = \left\lfloor \frac{n+4k}{3} \right\rfloor = \underline{\underline{\left\lfloor \frac{n+2m}{3} \right\rfloor}}$$

ODD M

$m < \left\lfloor \frac{2n}{5} \right\rfloor$

$m = 2k+1$
$n - 5k - 1$

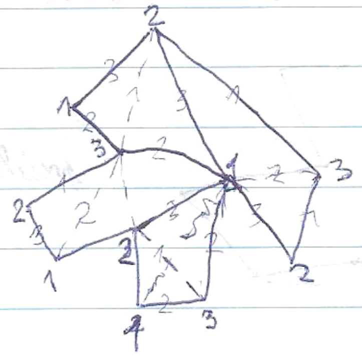


$$G = \left\lfloor \frac{n-5k-1}{3} \right\rfloor + 1 + 3k = \left\lfloor \frac{n+4k+2}{3} \right\rfloor =$$

$$= \underline{\underline{\left\lfloor \frac{n+2m}{3} \right\rfloor}}$$

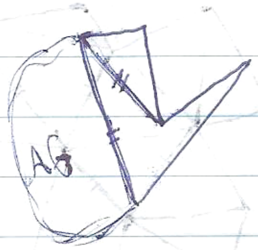
② OBSTRUCTORS

- triangulation, 3-col. vertexes, then 3-colouring edges (~~the~~ colour of the opposite vertex)
- ~~great~~ best approach for high number of IW



no. of edges

$$\left\lfloor \frac{2n-3}{3} \right\rfloor \qquad m \geq \left\lfloor \frac{2}{3} n \right\rfloor - 2$$



V +3

IW +2

G +2

$k = \left\lfloor \frac{n}{3} \right\rfloor - 1$

$V = 3 + \left(\left\lfloor \frac{n}{3} \right\rfloor - 1\right) \cdot 3 = 3 + n - 3$

$m = \left(\left\lfloor \frac{n}{3} \right\rfloor - 1\right) \cdot 2 = \left\lfloor \frac{2n}{3} \right\rfloor - 2$

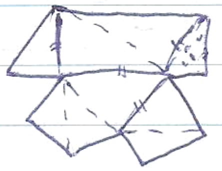
$G = 1 + \left(\left\lfloor \frac{n}{3} \right\rfloor - 1\right) \cdot 2 = 1 + \left\lfloor \frac{2n}{3} \right\rfloor - 2 = \underline{\underline{\left\lfloor \frac{2n-3}{3} \right\rfloor}}$

$n = 3k$ △

$n = 3k+1$ □

$n = 3k+2$

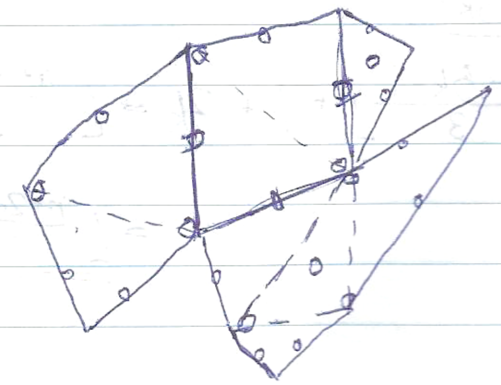
③ KARMA POINTS – approach for m between $\left\lfloor \frac{2}{3} n \right\rfloor - 2$ and $\left\lfloor \frac{2n}{5} \right\rfloor$

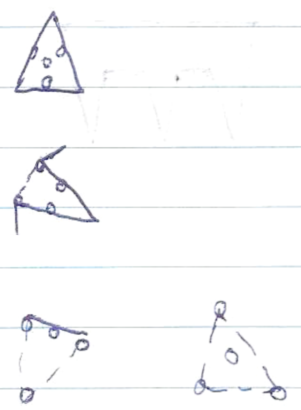


X

1. $\forall x \in X$ guards all incident faces
2. each △ is guarded exactly by 4 x
3. X can be 4 coloured in a special way
4. $|X| = 2n + m - 2$

$$G \leq \left\lfloor \frac{2n + m - 2}{4} \right\rfloor$$





$|X| = W + T + C$

$W: \; m + n$

DUAL GRAPH

$AG \rightarrow$ TREE

$T: \; t$

$C: \; n-2-t$

$n-1 \quad E$ edges

$2n-2 \quad \Sigma d(V)$ degrees

$a \cdot 1 + b \cdot 2 + c \cdot 3 = 2(a+b+c) - 2$

$c + 2 = a$

$\text{DIVIDED}_j = r_j - 2 - t_j$

vertices

no. of rooms

$$\sum_{j=1}^{m+1} r_j - 2 - t_j = -t - 2(m+1) + n + 2m =$$

$$= n - 2 - t$$

$|X| = m + n + t + n - 2 - t =$

$= \underline{2n + m - 2}$

$\left\lfloor \frac{n+2m}{3} \right\rfloor G \qquad m < \left\lfloor \frac{2n}{5} \right\rfloor$

$\left\lfloor \frac{2n-3}{3} \right\rfloor G \qquad m \geq \left\lfloor \frac{2}{3} n \right\rfloor - 2$

$\left\lfloor \frac{2n+m-2}{4} \right\rfloor G \qquad$ otherwise