MA051

Arc

continous, injective $[0,1] \to \mathbb{R}^2$

Arc-wise connectivity

Connectivity

- cannot be covered by two disjoint open sets

Compactness

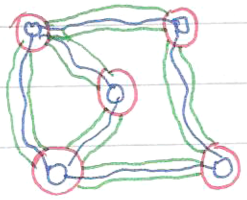
-

$\forall f$: compact $\to \mathbb{R}$ has maximum and minimum

Look up

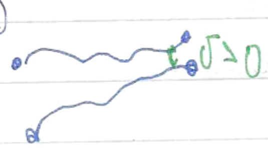
open set
basics of topology
preimage of an open set
topological continuity
topological connectivity
boundary

Can every planar embedding of a graph stretch so it use only polygonal edges



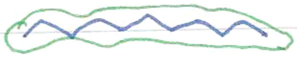
3 problems

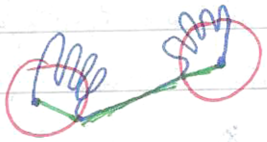
1. 

2. 

- the number of intersections may be infinite
- arc may share a part of boundary

- minimal distance between arcs is bigger than 0

3. 

1. 

3. **Lemma:** If Ω is an open arcwise connected set in $\mathbb{R}^2$, then any 2 points $x, y \in \mathbb{R}$ are connected by a polyline in Ω.

2. **Lemma:** If a, b are disjoint arcs in $\mathbb{R}^2$ then distance between $a, b > 0$.

Def: An arc in $\mathbb{R}^2$ is the image of a suitable continuous function $a: [0,1]$

The ends of a arc $a(0), a(1)$.

- the function $B \to \mathbb{R}$ that assignes distance of point from B to A_1 has its minimum, say it is achieved in B_1
- analogical function $A \to \mathbb{R}$ has minimum in A_2
- if $|A_2 B_1| = 0 \Rightarrow A_2 = B_1 \Rightarrow$ this point belongs to both sets

Theorem$^{(1)}$: If a graph G is embedded in $\mathbb{R}^2$, then there is a polyline embedding in $\mathbb{R}^2$.

Definition: An embedding of a graph G in a topological space Ω is a ~~one~~ injective mapping $V(G) \to \Omega$
+ a mapping simple (simple = non-intersecting) arcs $\to \Omega$ such that:

1) $\forall\, uv \in E(G)$, the arc uv has ends u and v
2) $\forall\, uv \in E(G)$, the interior of the arc uv is disjoint from the rest of the embedding

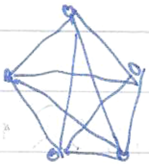
The aspects of topological graph theory

Historical	Modern
- Euler's formula	- algorithms on planar graphs
- 4 color theorem	- crossing number
- regular MAPS	- graph minors
- Kuratowski theorem	

Outline of a proof of ~~a~~ Kuratowski's theorem

Jordan curve theorem on polygons is easy $\Rightarrow$ you have Euler's formula

- Why K_5 is non-planar?
  - by (1) it has polyline embedding
  - now we have Euler's formula



$$\left.\begin{matrix} e = 10 \\ \cancel{f=}\, v = 5 \end{matrix}\right\} \Rightarrow f = e - v + 2 = 7$$

$$\Downarrow$$

it has at least $f \cdot 3$ edges > 10

⋮