Jordan Curve Theorem and Euler's formula

THM: Every simple closed curve in $\mathbb{R}^2$ divides $\mathbb{R}^2$ into two arcwise connected regions ($\mathbb{R}^2 \setminus C$).
THM: Euler's formula

Lemma 1: JCT $\Leftarrow\Rightarrow$ EUL (proof $\Rightarrow$ in MA010)

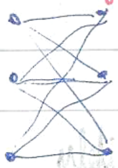
Substatement of a Jordan Curve Theorem: No simple curve divides $\mathbb{R}^2$ into > 1 arcwise connected regions.

$\Leftarrow$ direction of L1:
- closed curve

3 vertices + 3 edges $\Rightarrow$ by EUL faces = 2 ✓

Lemma 2: JCT holds for polygonal C.
Proof: Orientate graph in a such a way that no line is horizontal.

Corollary 1: $K_{3,3}$ is not planar

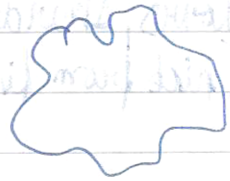


$v = 6$
$e = 9$
$f =$

Choose two points near interior of an arbitrary edge
one of them intersects curve odd number of times
even number of times $\Rightarrow$ at least 2 regions

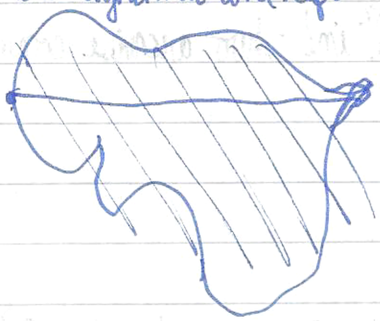
- why at most 2 regions?
- choose 3 representatives, stop before hitting an edge and continue along the edge $\Rightarrow$ in some time you'll get to those 2 points

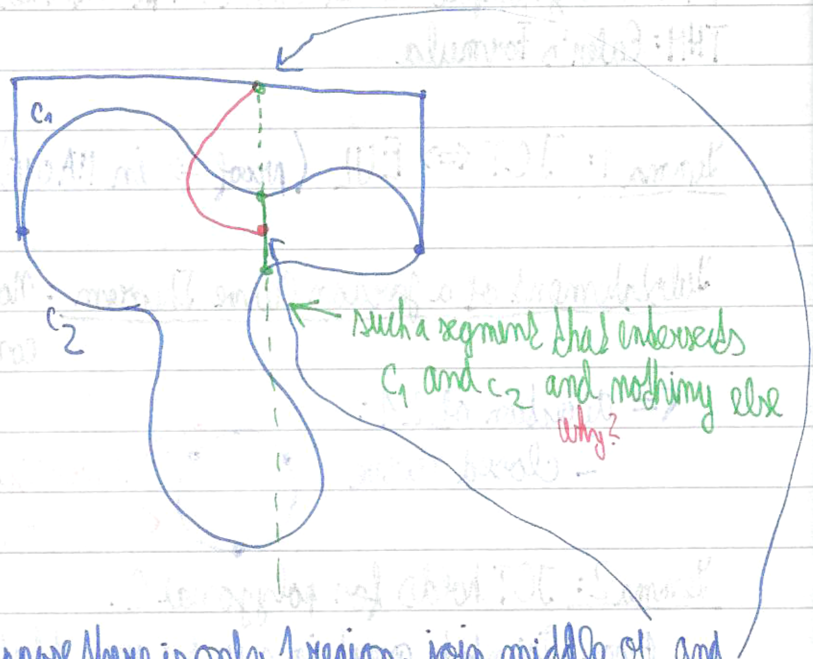
THM: JCT $\Leftarrow\Rightarrow$ "$K_{3,3}$ is not planar" (in any topological space)
1.) simple closed curve C, then $\mathbb{R}^2 \setminus C$ is not arcwise-connected (or $K_{3,3}$ is planar)



$C \equiv \gamma : [0,1] \to \mathbb{R}^2$
compact, continuous

- choose the rightmost and leftmost point (exists by compactness as intersection with $y=x$)



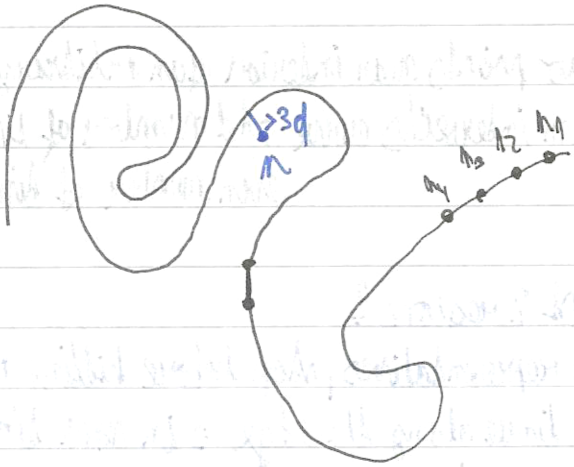


- we can now, by contradiction suppose there is only 1 region, join middle of and

→ $K_{3,3}$ appears!

2.) some technical lemma

3.) simple path P, then $\mathbb{R}^2 \setminus P$ is arcwise connected



- we claim that we can pick finitely many points such that distance of every point of P between r_i and r_{i+1} is less than d

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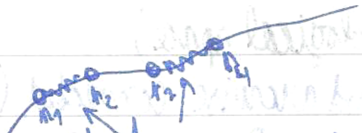
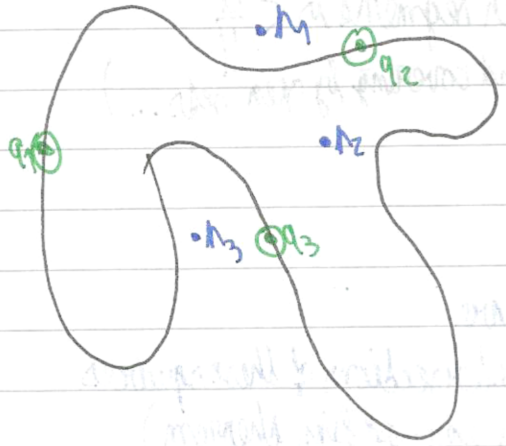


if two segments are not neighbours, their distance is some $d>0$

- we take minimal distance d' (pick from finitely many)

4.) simple closed curve C, then $\mathbb{R}^2 \setminus C$ has ≤ 2 arcwise-connected components (or you can draw $K_{3,3}$)

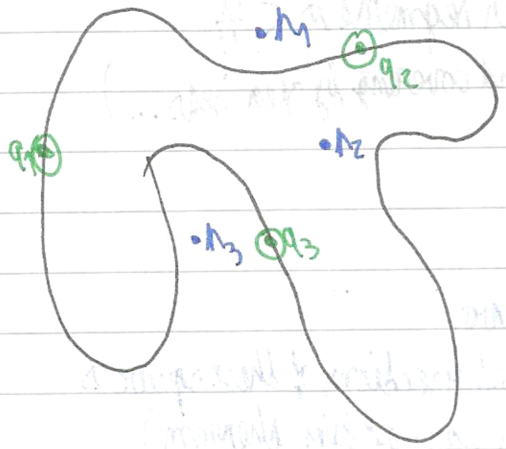


– suppose there are at least 3 mos arc-con. regions p_1, p_2, p_3

– chose any 3 points on the curve q_1, q_2, q_3

finish!

4.) simple closed curve C, then $\mathbb{R}^2 \setminus C$ has ≤ 2 arcwise-connected components (or you can draw $K_{3,3}$)



– suppose there are at least 3 not arc-con. regions p_1, p_2, p_3

– chose any 3 points on the curve q_1, q_2, q_3

finish!