lin. zobrazení: $f: V \to W$

$f(u +_V v) = f(u) +_W f(v)$

$f(a \cdot v) = a \cdot f(v)$

prostory se skal. součinem

$u, v \in V: \langle u, v \rangle \in \mathbb{R}$

$V \times V \to \mathbb{R}$

ortog. zobr.: $f: V \to W$

$\forall u, v \in V: \langle u, v \rangle_V = \langle f(u), f(v) \rangle_W$

4 30-9:56

$\langle f(u+v), f(u+v) \rangle = \langle f(u) + f(v), f(u) + f(v) \rangle =$

$= \langle f(u), f(u) + f(v) \rangle + \langle f(v), f(u) + f(v) \rangle =$

$= \langle f(u), f(u) \rangle + \langle f(u), f(v) \rangle + \langle f(v), f(u) \rangle + \langle f(v), f(v) \rangle$

ortog. zobr. $\langle f(v), f(v) \rangle = \langle v, v \rangle$

$= \langle u, u \rangle + \langle v, v \rangle + 2 \langle f(u), f(v) \rangle$

Jinak: $\langle f(u+v), f(u+v) \rangle = \langle u+v, u+v \rangle =$

$= \langle u, u \rangle + \langle v, v \rangle + 2 \langle u, v \rangle$

$\Rightarrow \langle f(u), f(v) \rangle = \langle u, v \rangle$

4 30-10:08

$R_\alpha: \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = A$

$\mathbb{R}^2 \to \mathbb{R}^2$

splňuje $A^T = A^{-1}$

4 30-10:14

lin. zobrazení f je injektivní $\Leftrightarrow$

$\Leftrightarrow \ker f = \{0\}$

„$\Rightarrow$" zřejmé

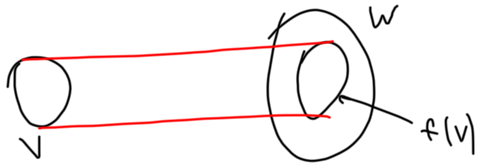
„$\Leftarrow$" f není injektivní: $\exists u, v: f(u) = f(v)$, $u \neq v$

$0 = f(u) - f(v) = f(u - v)$

$u - v \neq 0$

4 30-10:16

$f: V \xrightarrow{\text{injektivní}} W$



ztotožňujeme (někdy) V a f(V)

($V \subset W$)

4 30-10:20

$f: V \to V$

ortonorm. ortonorm. bázi $\underline{x}$

$f(u) = A[u]_x^T$

4 30-10:22

$$(A\cdot x)^T\cdot(A\cdot y) = x^T\cdot(A^T\cdot A)\cdot y = x^T\cdot y.$$

$$\langle f(x), f(y)\rangle = \langle x, y\rangle$$

f je ortog. zobr.

$\boxed{} \Rightarrow A^T \cdot A = E$

$$(0 \dots 1 \dots 0)\,(A^T\cdot A)\begin{pmatrix}0\\1\\0\end{pmatrix} = (A^T\cdot A)_{ij}$$

$x^T \cdot y =$ (pro x, y vektory orton. báze) $= \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$

$$(A^T\cdot A)_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \Rightarrow A^T\cdot A = E$$

4 30-10:24

4 30-10:31

Věta

Nechť V je reálný vektorový prostor se skalárním součinem a $f : V \to V$ je lineární zobrazení. Pak f je ortogonální, právě když v některé ortonormální bázi (a pak už ve všech) má matici A splňující $A^T = A^{-1}$ (tedy ortogonální matici).

2 báze V, kde matice f vzhl. k bázím jsou A, B, $\exists!$ ex. P (regulární):

$$A = P^{-1}\cdot B\cdot P$$

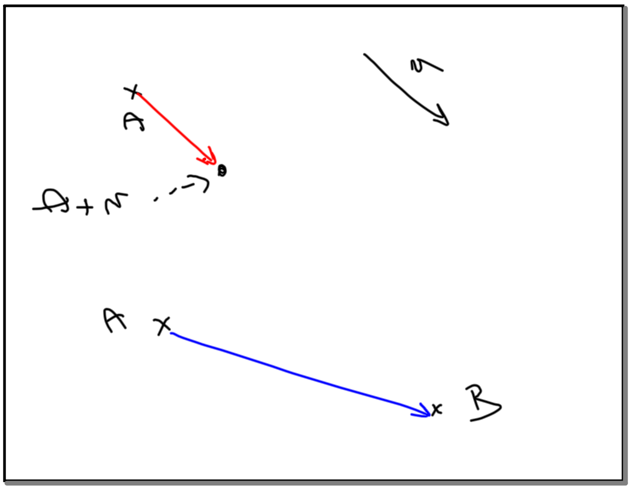
$$A^T = A^{-1} \Rightarrow B^T = (P\cdot A\cdot P^{-1})^T = (P^{-1})^T\cdot A^T\cdot P^T = (P^{-1})^T\cdot A^{-1}\cdot P^T =$$

báze orton.

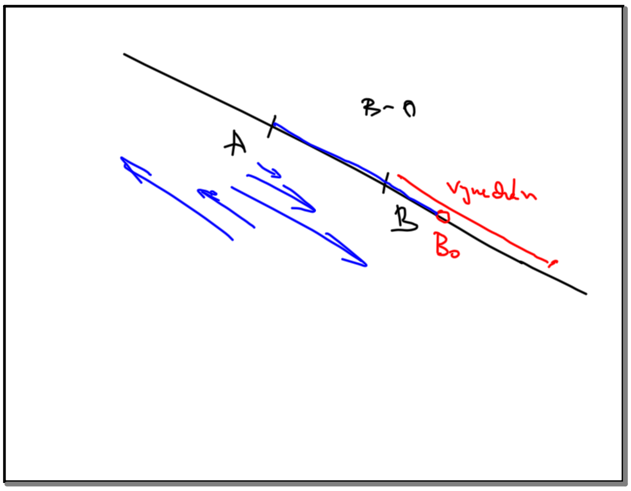
$$(P^T)^{-1}\cdot A^{-1}\cdot P^{-1} = (PAP^{-1})^{-1} = B^{-1}$$

$$(P^{-1})^T = (P^T)^{-1}$$

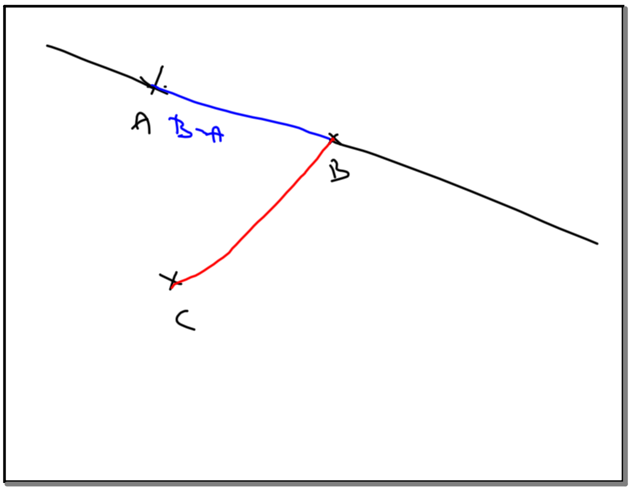
4 30-10:31



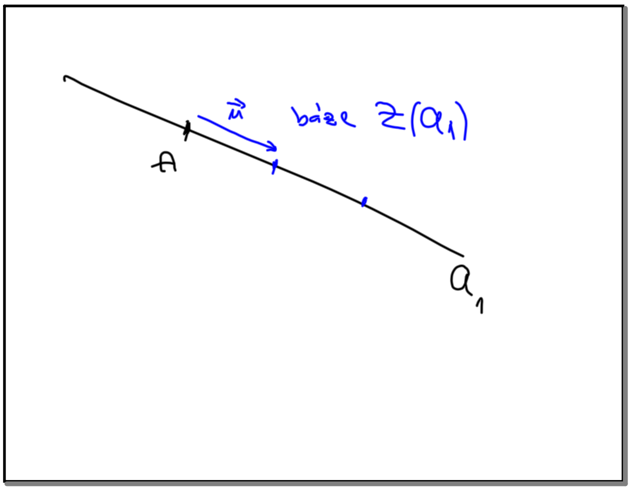
4 30-10:31



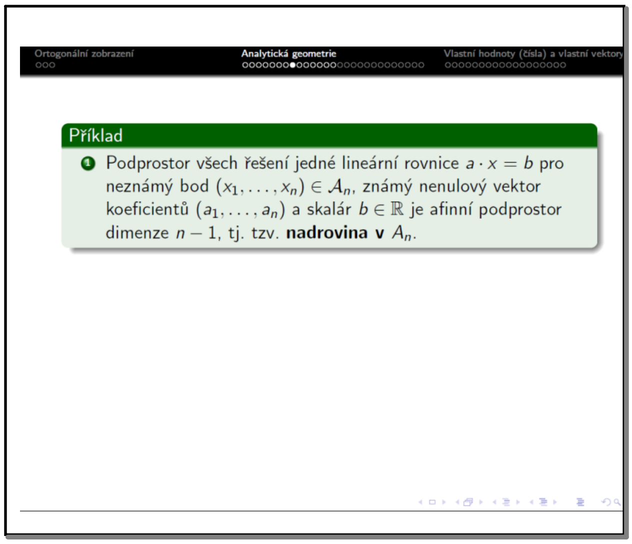
4 30-10:48



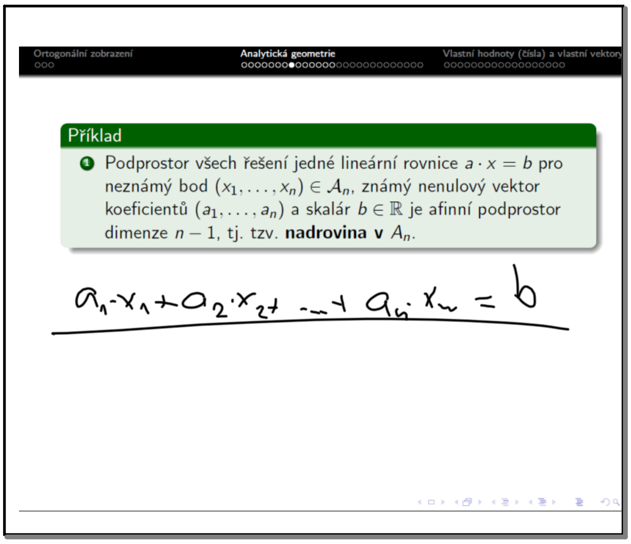
4 30-10:51



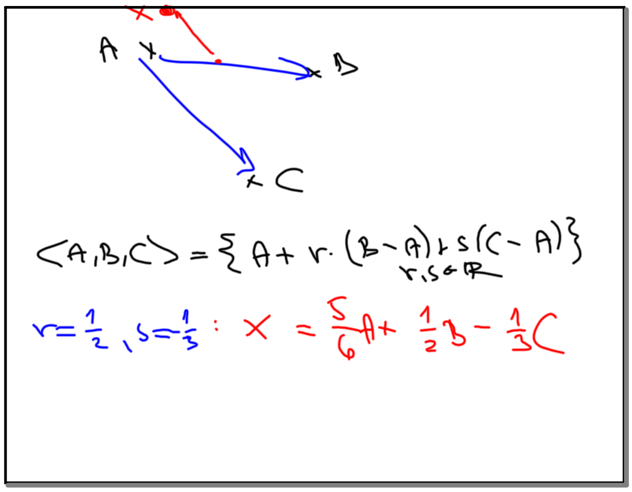
4 30-10:58



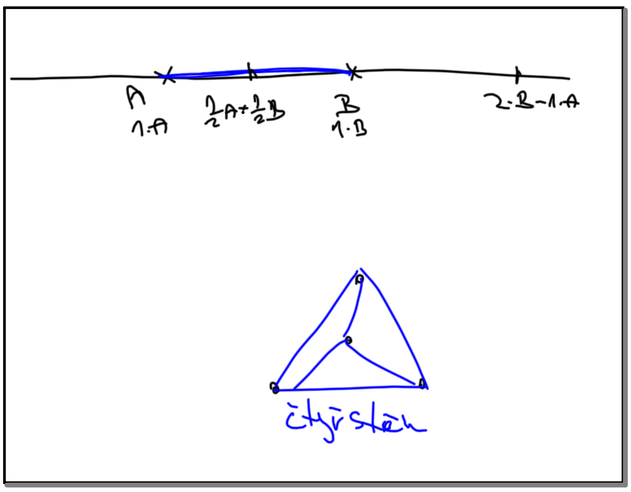
4 30-11:01



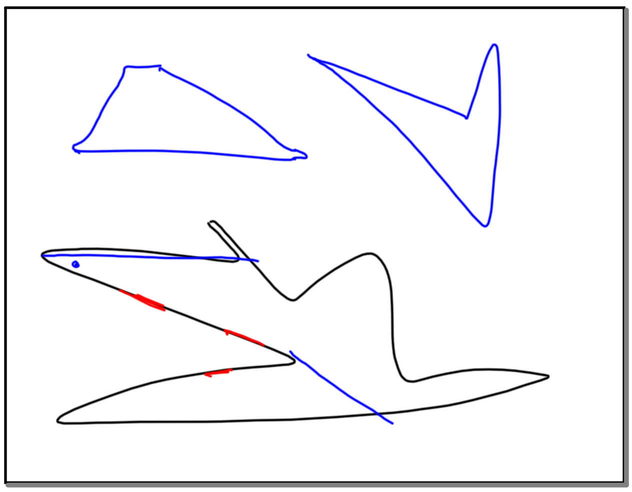
4 30-11:01



4 30-11:03



4 30-11:16



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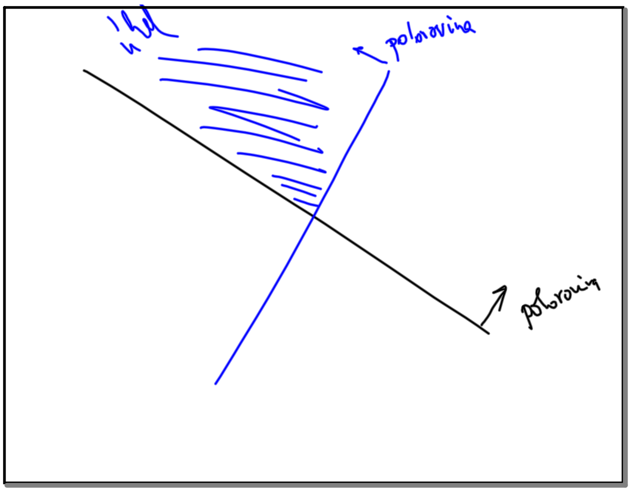
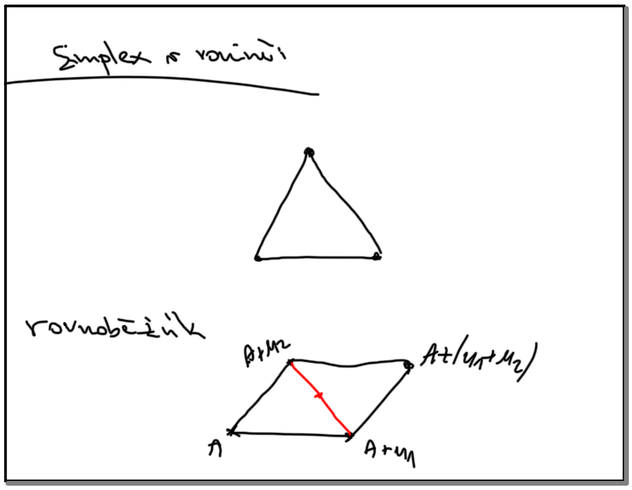
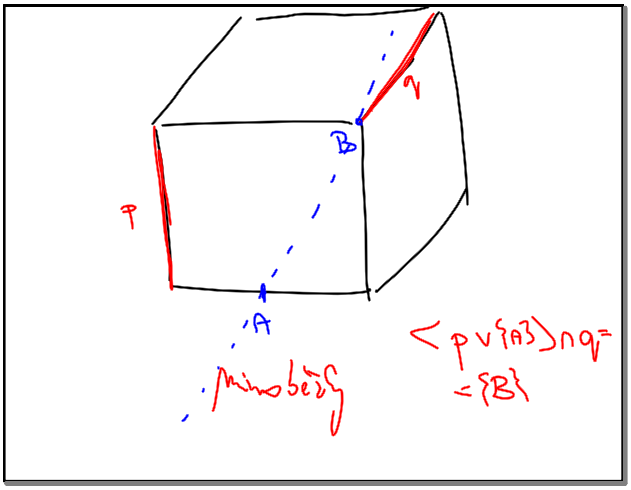


4 30-11:24



4 30-11:28



4 30-11:37