$$\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & & \\ a_{m1} & \cdots & a_{mm} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

matice soustavy

$$\left(A \,\middle|\, \begin{matrix} y_1 \\ \vdots \\ y_m \end{matrix}\right)$$

rozš. matice soustavy

$a_i \cdot X = y_i \qquad i = 1, \dots, m$

$$a_{i1} \cdot x_1 + a_{i2} \cdot x_2 + \cdots + a_{im} x_m = y_i$$

$$A \cdot x^T = y^T \qquad / \cdot A^{-1}$$

$$x^T = A^{-1} \cdot y^T$$

3 19-10:18

$$_m\begin{pmatrix} & & \\ & & \end{pmatrix}^n \cdot {}_n\begin{pmatrix} & & \\ & & \end{pmatrix}^q$$

$m/n \qquad n/q$

$\Longrightarrow$ matice m/q

3 19-10:22

$$\underset{m/n}{A} \cdot \underset{n/q}{B} = \underset{m/q}{C}$$

$$c_{ik} = a_i \cdot b^k$$

i-tý řádek A

k-tý sloupec B

3 19-10:25

ex. e (jednotky)

(O3): $a \cdot e = e \cdot a = a \quad \forall a \in K$

Pro matice (čtvercové) je

$$e = E_n$$

$$A \cdot E_n = E_n \cdot A = A$$

$$A \cdot \begin{pmatrix} 1 & \cdots & 0 \\ & 1 & \\ 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} a_i \cdot e^j \end{pmatrix} = \begin{pmatrix} a_{ij} \end{pmatrix}$$

$e^j = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ ← j-tá

3 19-10:29

nekomutativita:

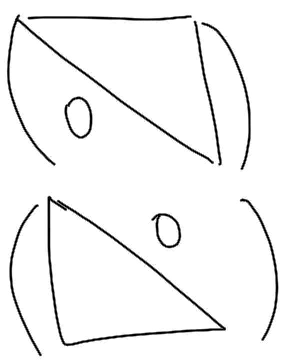
$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \quad \#$$

dělitelé nuly:

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

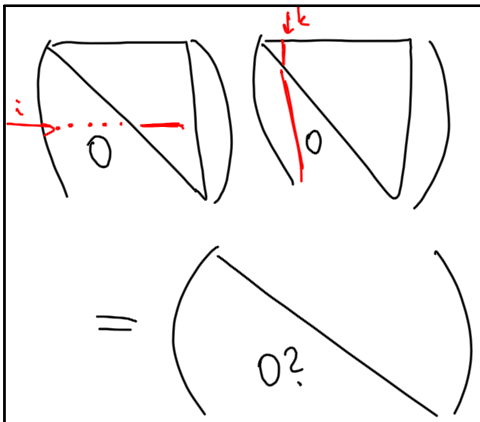
3 19-10:33



horní Δ

dolní Δ

3 19-10:39



Pro c_{ik}, $i>k$ platí $c_{ik}=0$

3 19-10:40

B inverzní k A $\iff$ $A\cdot B = B\cdot A = E$

$(B = A^{-1})$

$\iff$ A je inverzní k B

$(A = B^{-1})$

$A = B^{-1} = (A^{-1})^{-1}$

3 19-10:45

jednoznačnost inverze:

B, C ... inverze k A:

$A\cdot B = B\cdot A = A\cdot C = C\cdot A = E$

$A\cdot B = A\cdot C \quad | \; B\cdot$

$(BA)\cdot B = (B\cdot A)\cdot C$

$E\cdot B = E\cdot C$

$B = C$

3 19-10:51

$A\cdot A^{-1} = E$

$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} = A$

$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$b_{11} + 3b_{21} = 1 \qquad b_{12} + 3b_{22} = 0$

$2b_{11} - b_{21} = 0 \qquad 2b_{12} - b_{22} = 1$

3 19-10:55

Dk: $(AB)^T = B^T\cdot A^T$

$((AB)^T)_{ij} = (AB)_{ji} = a_j \cdot b^i = a_{j1}b_{1i} + a_{j2}b_{2i} + \dots$

řádek j, sloupec i

$(B^T\cdot A^T)_{ij} = (B^T)_i \cdot (A^T)^j = b^i \cdot a_j =$

řádek i, sloupec j

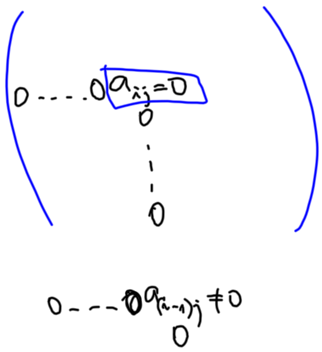
$= b_{1i}\cdot a_{j1} + b_{2i}\cdot a_{j2} + \dots$

$(A^T)^{-1} = (A^{-1})^T$

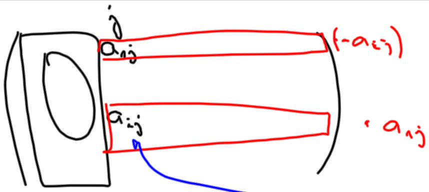
$A^T\cdot(A^{-1})^T = (A^{-1}\cdot A)^T = E^T = E$

$(A^{-1})^T\cdot A^T = (A\cdot A^{-1})^T = E^T = E$ ✓

3 19-11:02



3 19-11:12



$$a_{ij}\cdot a_{1j} + (-a_{ij})\cdot a_{1j} = 0$$

3 19-11:16

$$\begin{pmatrix} 2 & & & \\ & 3 & & \\ & & 4 & \\ & & & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 9 \\ 2 \end{pmatrix}$$

$2\cdot x_1 = 4$ mod $\mathbb{Z}$

$3x_2 = 6$

$4\cdot x_3 = 9$ $x_3 \notin \mathbb{Z}$

3 19-11:18

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}$$

$2 \leftrightarrow 4.$

$$= \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{41} & b_{42} & b_{43} & b_{44} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{pmatrix}$$

3 19-11:21

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} =$$

$a_{23} = 1$

$$= \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{21}+b_{31} & b_{22}+b_{32} & b_{23}+b_{33} \end{pmatrix}$$

Tj. přičtení 2. řádku k 3.

3 19-11:24

$$(A \mid E)$$

$$\wr$$

$P_k \cdots P_1 \cdot A = E$ $P_k \cdots P_1 \cdot E = A^{-1}$

$$(E \mid A^{-1})$$

3 19-11:34

3 19-11:41