skal. součin:

$\mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$

(obecně:

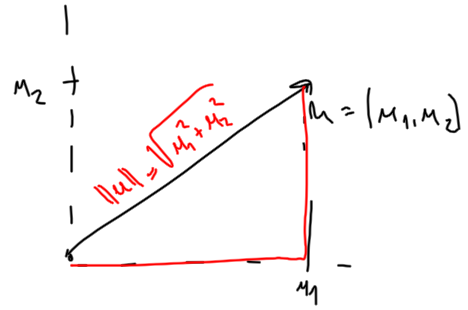
$V \times V \longrightarrow \mathbb{K}$)

násobení vektoru skalárem

$\mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$

$(\mathbb{K} \times V \to V)$

4 23-9:53



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Cauchyova nerovnost:

$(x_1, \dots, x_n)$, $(y_1, \dots, y_n) \in \mathbb{R}^n$

$$\left(\sum_{i=1}^{n} x_i y_i\right)^2 \leq \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right)$$

$$(x_1y_1 + \dots + x_ny_n)^2 \leq (x_1^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2)$$

myšlenka Dk: $t \in \mathbb{R}$ pomocná neznámá

$f(t) = (x_1 + t \cdot y_1)^2 + \dots + (x_n + t \cdot y_n)^2$

$f(t) = (x_1^2 + 2t x_1 y_1 + t^2 y_1^2) + \dots + (x_n^2 + 2t x_n y_n + t^2 y_n^2)$

$f(t) = t^2(y_1^2 + \dots + y_n^2) + 2t(x_1y_1 + \dots + x_ny_n) + (x_1^2 + \dots + x_n^2)$

$f(t) = 0 \iff x_1 + t \cdot y_1 = 0$
$\vdots$
$x_n + t \cdot y_n = 0$

Proto diskriminant $D \leq 0$.

$D = \text{"}b^2 - 4ac\text{"} = 4(x_1y_1 + \dots + x_ny_n)^2 - 4(y_1^2 + \dots + y_n^2)(x_1^2 + \dots + x_n^2)$

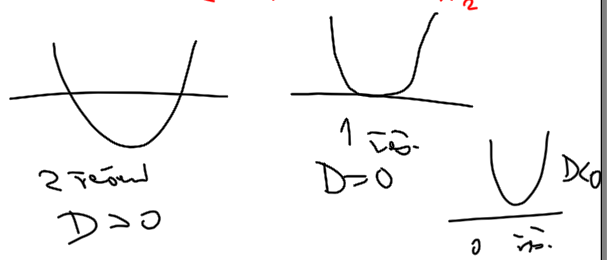
$D \leq 0 \iff (x_1y_1 + \dots + x_ny_n)^2 \leq (x_1^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2)$

4 23-10:09

$x^2 = 0$ 1 kořen $x = 0$

$(x+2)^2 = 0$ 1 kořen $x = -2$

$(2x_1 + 3)^2 + (x_2 - 4)^2 = 0$ $x_1 = -\frac{3}{2}$, $x_2 = 4$



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obecná rovnice roviny v $\mathbb{R}^3$:

$\rho: 2x_1 - x_2 + 3x_3 = 4$

norm. vektor $(2, -1, 3)$

je kolmý na všechny vektory roviny ρ

4 23-10:18

Dk věty (i) $x \in X \cap Y$, $x \neq \mathbf{o}$

$\Rightarrow x \not\perp x \Rightarrow X \not\perp Y$

(ii) chceme, že $Y^\perp$ je v. p.

$u, v \in Y^\perp \Rightarrow u + v \in Y^\perp$

$a \in \mathbb{R}$ $\quad \underline{a \cdot u} \in Y^\perp$

chci: $\forall y \in Y: \langle a \cdot u, y \rangle = 0$

$\overset{\text{sk. součin}}{=}$

$a \cdot u_1 \cdot y_1 + a \cdot u_2 \cdot y_2 + \dots + a \cdot u_n \cdot y_n =$

$= a \cdot \underbrace{(u_1y_1 + \dots + u_ny_n)}_{= 0 \text{, neboť } \langle u, y \rangle = 0} = 0$

chci: $\forall y \in Y: \langle u + v, y \rangle = 0$

$(u_1 + v_1) \cdot y_1 + \dots + (u_n + v_n) y_n =$

$= \underbrace{(u_1y_1 + u_2y_2 + \dots + u_ny_n)}_{= 0} + \underbrace{(v_1y_1 + \dots + v_ny_n)}_{= 0}$

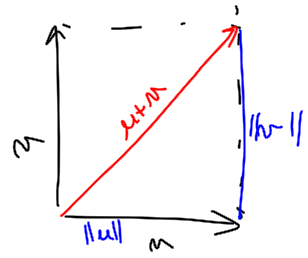
4 23-10:35

skal. součin na $\mathbb{R}^n$

$x_1y_1 + 2x_2y_2 + 3x_3y_3 + \dots + n x_n y_n$

4 23-10:46

Pythagorova věta v $\mathbb{R}^2$:



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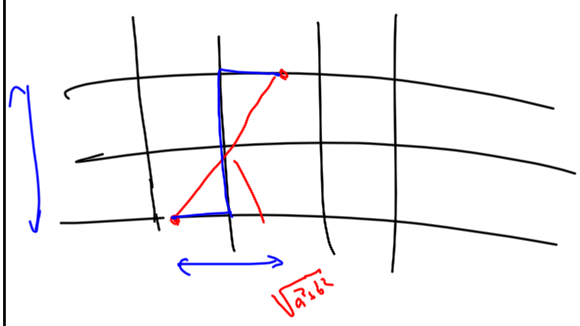

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$\langle o, u \rangle \overset{?}{=} 0$ ✓

linearita:

$\langle 0\cdot v, u \rangle = 0 \cdot \langle v, u \rangle$
$= 0$

$v \neq 0$

4 23-11:07

$\mathbb{R}^n$: $e_1 = (1, 0, \dots, 0)$
$e_2 = (0, 1, \dots, 0)$
$\vdots$
$e_n = (0, 0, \dots, 1)$

vektor $v = (\underline{1}, \underline{2}, \dots, \underline{n})$
souřadnice

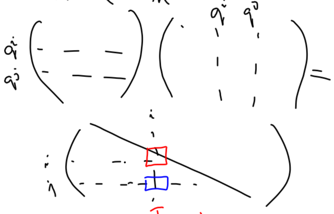
$v_i = \langle v, e_i \rangle = i$

4 23-11:16

Q ortogonální matice

sloupce ortonormální!

$Q^T \cdot Q = E_n$



$\square = 1 \Leftrightarrow (q^i)^T \cdot q^i = 1$

$\square = 0 \Leftrightarrow (q^j)^T \cdot q^i = 0$

4 23-11:19

odchylka v od W je φ, kde:

$(v = p + m)$

$$\cos\varphi = \frac{\langle v, p\rangle}{\|v\|\cdot\|p\|} = \frac{\|p\|^2}{\|v\|\cdot\|p\|} = \frac{\|p\|}{\|v\|}$$

4 23-11:35