

$x + ay - cz = 0$   
 $x + (a+b)y + cz = 5$   
 $x + (a+b)y + 2z = d - 1$

$\begin{pmatrix} 1 & a & -c & 0 \\ 1 & a+b & c & 5 \\ 1 & a+b & 2 & d-1 \end{pmatrix} \sim \begin{pmatrix} 1 & a & -c & 0 \\ 0 & b & c & 5 \\ 0 & b & 2+c & d-1 \end{pmatrix}$

$\begin{pmatrix} 1 & a & -c & 0 \\ 0 & b & c & 5 \\ 0 & 0 & 2 & d-1-5 \end{pmatrix} \sim \begin{pmatrix} 1 & a & -c & 0 \\ 0 & b & c & 5 \\ 0 & 0 & 2 & d-6 \end{pmatrix}$

$(1-c)z = -d$   
 $z = \frac{-d}{1-c}$

$y + bz = \frac{5}{b}$   
 $y = \frac{5}{b} - bz$

$x + ay - cz = 0$   
 $x = -ay + cz$

$\Rightarrow \left\{ \frac{5}{b} - bz, \frac{5}{b} - bz, \frac{-d}{1-c} \right\}$

ii)  $d=0$   
 $0z = d$   
 $0 = d$

iii)  $c=1$   
 $(1-c)z = -d$   
 $0z = -d$   
 $0 = -d$

$z = \frac{5}{b}$   
 $y = \frac{5}{b} - bz$   
 $x = -ay + cz$

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$ax + by = c$   
 $ax + by + az = b$   
 $ay + bz = a$

$L(A) = L(A|B) \Rightarrow \exists \text{ řešení}$

$\det A = 3$   
 $L(A) = 3$      $L(A|B) = 3$

$\begin{vmatrix} a & b & 0 \\ c & 0 & a \\ 0 & c & b \end{vmatrix} = -cb^2 - a^2c = -c(a^2 + b^2) = 0$   
 $\Leftrightarrow c = 0$   
 $\vee (a^2 + b^2) = 0$   
 $\Leftrightarrow a = b = 0$

$\exists \text{ řešení} \Leftrightarrow (a,b,c) \in \{(s,t,0), (0,0,r)\}$

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$ax + by + z = 1$   
 $x + ay + z = b$   
 $x + by + az = 1$

$\begin{pmatrix} a & b & 1 & 1 \\ 1 & a & 1 & b \\ 1 & b & a & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & a & 1 & b \\ 0 & b-a & 0 & 1-b \\ 0 & b-a & a-1 & 1-b \end{pmatrix}$

$(b-a)z = 1-b$   
 $z = \frac{1-b}{b-a}$

$y + bz = \frac{1-b}{b-a}$   
 $y = \frac{1-b}{b-a} - bz$

$x + ay + z = b$   
 $x = b - ay - z$

$\Rightarrow \left\{ \frac{1-b}{b-a} - bz, \frac{1-b}{b-a} - bz, \frac{1-b}{b-a} \right\}$

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$\begin{matrix} a & b & k \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{matrix} \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix} \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix}$

$\begin{pmatrix} 1 & b & a & 1 \\ 0 & b-a & a-1 & 1-b \\ 0 & a-a & a-a & a-b \end{pmatrix}$

$z = \frac{a-b}{a^2+a-2}$

$(b-a)y = 1-b - \frac{(a-1)(a-b)}{a^2+a-2}$

$b(1-a)y = 1-b - \frac{a-b}{a+2}$

$b=0$   
 $0 = 1 - \frac{a}{a+2}$

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$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & a & ab \\ b & a^2 & a^2b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & 0 & a^2-ab \\ b & a^2 & a^2b \end{vmatrix}$

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$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \frac{bc^2 + ab^2 + a^2c - a^2b - ab^2c - a^2c^2}{(a-b)(b-c)(c-a)}$

Vandermonde determinant

$\begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 & \dots \\ 1 & x_2 & x_2^2 & x_2^3 & \dots \\ 1 & x_3 & x_3^2 & x_3^3 & \dots \\ 1 & x_4 & x_4^2 & x_4^3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{vmatrix}$

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$$\cdot 4 \left( \begin{array}{cccc|ccc} 6 & 3 & 2 & 3 & 4 & 0 & 1 & 2 \\ 4 & 2 & 1 & 2 & 3 & 0 & 1 & 3 \\ 4 & 2 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 7 & 3 & 2 & 0 & 1 & 3 \end{array} \right) \sim$$

$$\left( \begin{array}{cccc|ccc} 6 & 3 & 2 & 3 & 4 & 0 & 1 & 2 \\ 0 & 0 & 2 & 0 & -2 & 0 & -2 & -10 \\ & & & & & & \uparrow & \uparrow \\ & & & & & & 0 & 0 \\ & & & & & & 0 & 0 \\ & & & & & & 0 & 0 \end{array} \right)$$

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$$\begin{pmatrix} 3 & 1 & -1 & 1 \\ 2 & -1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -2 + 1 - 1 - 5 = -5$$

$$|A_1| = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1 + 1 - 1 - 1 = -2$$

$$x = \frac{|A_1|}{|A|} = \frac{-2}{-5} = \frac{2}{5}$$

$$|A_2| = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -2 + 1 + 1 - 3 = -3$$

$$y = \frac{-3}{-5} = \frac{3}{5}$$

$$|A_3| = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -3 + 2 + 1 - 2 - 3 = -4$$

$$z = \frac{4}{5}$$

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$$\begin{pmatrix} 1 & 2 & 1 & -1 \\ 1 & 4 & 2 & 1 \\ 1 & 2 & 3 & 0 \\ 0 & 1 & 3 & 5 \end{pmatrix} = 3$$

$$|A_1| = \begin{vmatrix} 1 & 2 & 1 & -1 \\ 2 & 4 & 2 & 1 \\ 1 & 1 & 3 & 3 \end{vmatrix} = 9$$

$$\begin{aligned} & \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \cdot (-1) + \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} \cdot \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} \cdot (-1) \\ & + \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \cdot (-1) + \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} \cdot \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \cdot (-1) \\ & + \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} \cdot (-1) + \begin{vmatrix} 2 & 0 \\ 3 & 3 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \cdot (-1) \end{aligned}$$

$$|A_2| = 3$$

$$|A_3| = 3$$

$$|A_4| = -3$$

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$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$X = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2a+c & 2b+d \\ a+c & b+d \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 4a+2c+10b+5d \\ 2a+c+5b+5d \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{aligned} 4a+10b+2c+5d &= 2 \\ 2a+5b+2c+5d &= 1 \\ 2a+4b+c+2d &= 2 \\ a+2b+c+2d &= 1 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 4 & 1 & 2 & 1 \\ 2 & 5 & 2 & 5 & 1 \\ 4 & 10 & 2 & 5 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 2 & -1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 0 & -2 & -5 & 0 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

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