

$(1, -1, 0, 2) \quad (2, 2, -1, 3) \quad (0, 1, 1, 0) \quad (3, 2, 0, 5)$
 $t_1(1, -1, 0, 2) + t_2(2, 2, -1, 3) + t_3(0, 1, 1, 0) + t_4(3, 2, 0, 5)$
 $(t_1 + 2t_2 + 3t_4, -t_1 + 2t_2 + t_3 + 2t_4, t_2 + t_3, 2t_1 + 3t_2 + 5t_4)$
 $t_1 + 2t_2 + 3t_4 = 0$
 $-t_1 + 2t_2 + t_3 + 2t_4 = 0$
 $t_2 + t_3 = 0$
 $2t_1 + 3t_2 + 5t_4 = 0$

$$\begin{pmatrix} 1 & 2 & 0 & 3 & | & 0 \\ -1 & 2 & 1 & 2 & | & 0 \\ 0 & 1 & 1 & 0 & | & 0 \\ 2 & 3 & 0 & 5 & | & 0 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 2 & 0 & 3 & | & 0 \\ 0 & 4 & 1 & 5 & | & 0 \\ 0 & 1 & 1 & 0 & | & 0 \\ 0 & -1 & 0 & -1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 3 & | & 0 \\ 0 & 1 & 1 & 0 & | & 0 \\ 0 & 0 & 3 & 3 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \end{pmatrix}$$

\Rightarrow LZ
 $\begin{pmatrix} 1 & 2 & 0 & 3 & | & 0 \\ 0 & 1 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$

(u_1, u_2, u_3, u_4)

4 4-17:52

$U = \langle (1+2x-x^2), (2x+x^2), (5+x^2) \rangle$
 $? 1+x+x^2 \in U$
 $\dim(u_1, u_2, u_3) = \dim(u_1, u_2, u_3, u_4)$
 $1+x+x^2 = t_1(1+2x-x^2) + t_2(2x+x^2) + t_3(5+x^2)$

$x^2: 1 = -t_1 + t_2 + t_3$
 $x: 1 = 2t_1 + t_2$
 $1: 1 = t_1 + 2t_2 + 5t_3$

$$\begin{pmatrix} 1 & 2 & 5 & | & 1 \\ 2 & -1 & 0 & | & 1 \\ -1 & 1 & 1 & | & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 5 & | & 1 \\ 0 & -5 & -10 & | & -1 \\ 0 & 3 & 6 & | & 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 2 & 5 & | & 1 \\ 0 & 5 & 10 & | & -1 \\ 0 & 0 & 0 & | & 7 \end{pmatrix} \Rightarrow 0t_3 = 7$$

NR

4 4-18:12

Mat₂(R)

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = t_1 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + t_2 \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + t_3 \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + t_4 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$n_1: 1 = t_1 + t_2 + t_3 + t_4$
 $n_2: 2 = t_2 + t_3 + t_4$
 $n_3: 3 = t_3 + t_4$
 $n_4: 4 = t_4$

$t_4 = -1$
 $t_3 = -1$
 $t_2 = -1$
 $t_1 = -1$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = -1 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 1 \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} - 1 \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} - 1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

4 4-18:19

$u_1 = \langle (1, 1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0), (3, 2, 0, 5) \rangle$
 $u_2 = \langle (1, -1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$
 $\dim u_1 + \dim u_2 = \dim(u_1, u_2)$

$u_1 = \langle (1, 1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$
 $u_2 = \langle (1, -1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$

$$\begin{pmatrix} 1 & 1 & 0 & 2 & | & 1 \\ 2 & 1 & -1 & 3 & | & 2 \\ 0 & 1 & 1 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 2 & | & 1 \\ 0 & -1 & -1 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 2 & | & 1 \\ 0 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & 0 \end{pmatrix}$$

$\Rightarrow U_1 + U_2 = \mathbb{R}^4$
 $\dim(U_1 + U_2) = 4$

$u_1 = \langle (1, 1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$
 $u_2 = \langle (1, -1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$

$$\begin{pmatrix} 1 & 1 & 0 & 2 & | & 1 \\ 2 & 1 & -1 & 3 & | & 2 \\ 0 & 1 & 1 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 2 & | & 1 \\ 0 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 1 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 2 & | & 1 \\ 0 & 1 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 2 \end{pmatrix}$$

$\dim U_1 = 3$

4 4-18:22

$u_1 = \langle (1, 1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$
 $u_2 = \langle (1, -1, 0, 2), (2, 2, -1, 3), (0, 1, 1, 0) \rangle$
 $u \in U_1 \cup U_2 \Leftrightarrow u = u_1 + u_2$
 $t_1(1, 1, 0, 2) + t_2(2, 2, -1, 3) + t_3(0, 1, 1, 0) = s_1(1, -1, 0, 2) + s_2(2, 2, -1, 3) + s_3(0, 1, 1, 0)$

$$\begin{pmatrix} 4t_1 + 2t_2 + 3t_3 - s_1 - 2s_2 & = & 0 \\ t_2 + t_3 + s_2 - 2s_3 - s_3 & = & 0 \\ -2t_1 - 2t_2 - 2t_3 + 2s_1 - s_2 & = & 0 \\ 6t_1 + 3t_2 + 4t_3 - 2s_1 - 3s_2 & = & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 3 & -1 & -2 & 0 & | & 0 \\ 0 & 1 & 1 & 1 & -2 & -1 & | & 0 \\ 0 & -2 & -2 & 0 & 1 & -1 & | & 0 \\ 0 & 6 & 3 & 4 & -2 & -3 & | & 0 \end{pmatrix} \sim$$

$$\begin{pmatrix} 4 & 2 & 3 & -1 & -2 & 0 & | & 0 \\ 0 & 1 & 1 & 1 & -2 & -1 & | & 0 \\ 0 & 0 & 1 & 2 & -3 & -2 & | & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & | & 0 \end{pmatrix} \sim$$

$$\begin{pmatrix} 4 & 2 & 3 & -1 & -2 & 0 & | & 0 \\ 0 & 1 & 1 & 1 & -2 & -1 & | & 0 \\ 0 & 0 & 1 & 2 & -3 & -2 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 3 & -1 & -2 & 0 & | & 0 \\ 0 & 1 & 1 & 1 & -2 & -1 & | & 0 \\ 0 & 0 & 1 & 2 & -3 & -2 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$-6s_1 - 2s_2 - 2s_3 = 0$

4 4-18:39

$\begin{pmatrix} a & b \\ c & -a \end{pmatrix}, \begin{pmatrix} x & y \\ z & -x \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & -a \end{pmatrix} + \begin{pmatrix} x & y \\ z & -x \end{pmatrix} = \begin{pmatrix} x+a & y+b \\ z+x & -x-a \end{pmatrix} \in U$

$t \cdot \begin{pmatrix} a & b \\ c & -a \end{pmatrix} = \begin{pmatrix} ta & tb \\ tc & -ta \end{pmatrix} \in U$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = a \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + c \cdot \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + d \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

4 4-18:48

$\mathbb{R}_n[x]$ $n \geq 2$

$U = \{f \in \mathbb{R}_n[x] \mid f(2012) = f(2011) = 0\}$

$f \in U \Rightarrow \exists g: f = \underbrace{(x-2012)(x-2011)}_{\text{st } g \leq n-2} \cdot g$

$g = \underbrace{a_{n-2}}_{n-1 \text{ koeficientů}} x^{n-2} + \underbrace{a_{n-3}}_{n-1} x^{n-3} + \underbrace{a_{n-4}}_{n-1} x^{n-4} + \dots + \underbrace{a_1}_{n-1} x + \underbrace{a_0}_{n-1}$

$\dim U = n-1$

$U = \langle x^{n-2}(x-2012)(x-2011), x^{n-3}(x-2012)(x-2011), \dots, x(x-2012)(x-2011), (x-2012)(x-2011) \rangle$

4 4-18:55

$V_1, V_2, V_3 \in V$
 $S = \{a_1 v_1 + \dots + a_n v_n \mid \sum a_i v_i = 0\}$

1) $S \neq \text{polynom } \mathbb{R}^3$
 $a \in S, b \in S$
 $a = (a_1, \dots, a_n), b = (b_1, \dots, b_n)$
 $a+b = (a_1+b_1, \dots, a_n+b_n)$
 $(a_1+b_1)v_1 + \dots + (a_n+b_n)v_n = (a_1v_1 + \dots + a_nv_n) + (b_1v_1 + \dots + b_nv_n) = 0 + 0 = 0$
 $\Rightarrow a+b \in S$

2) $(a_1, \dots, a_n) \in S \Rightarrow a_1v_1 + \dots + a_nv_n = 0$
 $(b_1, \dots, b_n) \in S \Rightarrow b_1v_1 + \dots + b_nv_n = 0$
 $\Rightarrow (a_1+b_1)v_1 + \dots + (a_n+b_n)v_n = 0$
 $\Rightarrow S \in \text{PP}$

$V_1, V_2, V_3 \in V \Rightarrow \dim S = 0$

3) $V_1, V_2, V_3 \in V \Rightarrow t_1v_1 + \dots + t_nv_n = 0$
 $\Rightarrow S = \{0\} \Rightarrow \dim S = 0$

4) $\dim S = 0 \Rightarrow S = \{0\}$
 $\Rightarrow t_1v_1 + \dots + t_nv_n = 0$
 $\Rightarrow \text{LN}$

4 4-19:01

$x+y+z=0$
 $y-z=0 \Rightarrow y=z$

$x+y=0$
 $x=-y$
 $\{(t, -t, t) \mid t \in \mathbb{R}\}$
 pp \mathbb{R}^3

$\begin{cases} a_1x_1 + a_2x_2 + \dots + a_nx_n = 0 \\ \vdots \\ a_{i-1}x_1 + a_{i-2}x_2 + \dots + a_{i-1}x_n = 0 \end{cases}$

$\begin{pmatrix} r_1 & \dots & r_n \\ s_1 & \dots & s_n \end{pmatrix} \quad \begin{pmatrix} r_1 & \dots & r_n \\ s_1 & \dots & s_n \end{pmatrix}$

$a_{i1}(r_1+s_1) + a_{i2}(r_2+s_2) + \dots + a_{in}(r_n+s_n) = 0$
 $a_{i1}r_1 + \dots + a_{in}r_n + a_{i1}s_1 + \dots + a_{in}s_n = 0$
 $\underbrace{a_{i1}r_1 + \dots + a_{in}r_n}_0 + \underbrace{a_{i1}s_1 + \dots + a_{in}s_n}_0 = 0$
 (t_1, \dots, t_n)
 $a_{i1}t_1 + \dots + a_{in}t_n = 0$
 $t(a_{i1} + \dots + a_{in}) = 0$

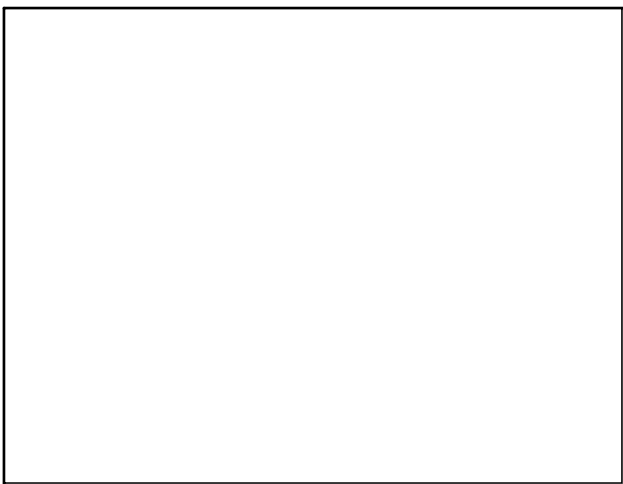
4 4-19:14

$(1, t, 6, 3) = s_1(1, -2, 2, 2) + s_2(1, -2, 2, 3) + s_3(1, 1, 0, 0)$

$s_1 + s_2 - s_3 = 2 \Rightarrow s_3 = 1$
 $-2s_1 - 2s_2 + s_3 = t$
 $2s_1 + 2s_2 = 6 \Rightarrow s_1 = 2$
 $3s_2 = 3 \Rightarrow s_2 = 1$

$-2s_1 - 2s_2 + s_3 = t$
 $-4 - 2 + 1 = t$
 $\boxed{-5 = t}$

4 4-19:25



4 4-19:29

$W_1 = \{x \in \mathbb{R}^3 \mid x = 2s_1 + 3s_2\}$
 $= \{2s_1 + 3s_2 \mid s_1, s_2 \in \mathbb{R}\}$
 $W_1 = \langle (2, 0, 0), (0, 3, 0) \rangle$
 $\dim W_1 = 2$

$W_2 = \{x \in \mathbb{R}^3 \mid x = 2s_1 - 3s_2\}$
 $W_2 = \langle (2, 0, 0), (0, -3, 0) \rangle$
 $\dim W_2 = 1$

$W_1 \cap W_2 = \{0\}$
 $W_1 + W_2 = \mathbb{R}^3$
 $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 = 2 + 1 = 3$

$W_1 \cap W_2 = \{0\}$
 $W_1 + W_2 = \mathbb{R}^3$
 $W_1 \oplus W_2 = \mathbb{R}^3$
 $W_1 \neq W_2$

$(1, 1, 1, 1) \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

4 4-19:29



4 4-18:34