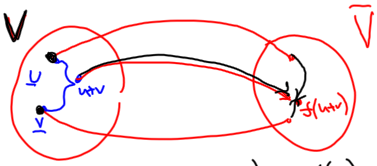
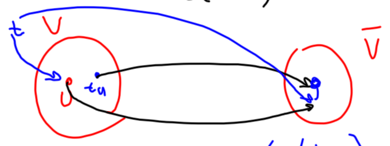
V $\bar{V}$



$$f(u+v) = f(u) + f(v)$$



$$f(t\cdot u) = t f(u)$$

$$\boxed{f(su+tv) = s\cdot f(u) + t\cdot f(v)}$$

$$f(1\cdot u - 1\cdot u) = f(\underline{0}) = f(1\cdot u) - f(1\cdot u) = 1 f(u) - 1 f(u) = \underline{0}$$

4 11-17:52

4 11-18:07

$f: \mathbb{R}^3 \to \mathbb{R}^2$

$f(a,b,c) = (1+a, b)$

$f(a,b,c) = (1,2)$

! $f(0_V) = 0_{\bar{V}}$!

$f((0,0,0)) = \underline{(1,0)}$

$f((a,b,c)) = (a+b, a-b)$

$f(u+v) = f(u) + f(v)$

$u = (u_1, u_2, u_3)$

$v = (v_1, v_2, v_3)$

$L = f(u+v) = f((u_1+v_1, u_2+v_2, u_3+v_3)) =$

$= (u_1+v_1+u_2+v_2, u_1+v_1-u_2-v_2)$

$P = f(u) + f(v) = f((u_1,u_2,u_3)) + f((v_1,v_2,v_3)) =$

$= (u_1+u_2, u_1-u_2) + (v_1+v_2, v_1-v_2)$

$(u_1+u_2+v_1+v_2, u_1-u_2+v_1-v_2)$

$f(t\cdot u) = t\cdot f(u)$

$= f(t(u_1,u_2,u_3)) = f((tu_1, tu_2, tu_3)) =$

$= (tu_1+tu_2, tu_1-tu_2)$

$P = t\cdot f(u) = t\cdot f((u_1,u_2,u_3)) =$

$= t(u_1+u_2, u_1-u_2)$

$= (tu_1+tu_2, tu_1-tu_2)$

LZ

jádro $f(a,b,c) = (0,0)$ $\underline{\mathrm{Ker} f \{(0,0,t) \mid t\in\mathbb{R}\}}$

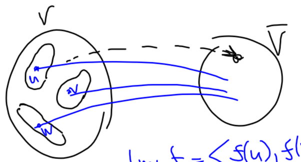
$(a+b, a-b) = (0,0)$

$a+b=0$ $a=0$

$a-b=0$ $b=0$

4 11-18:05

V $\bar{V}$



$\underline{\mathrm{Im} f = \langle f(u), f(v), f(w) \rangle}$

$\mathbb{R}^3 = \langle (1,0,0), (0,1,0), (0,0,1) \rangle$

$f((1,0,0)) = (1,1)$

$f((0,1,0)) = (1,-1)$

$f((0,0,1)) = (0,0)$

$\mathrm{Im} f = \langle (1,1), (1,-1), (0,0) \rangle$

$= \mathbb{R}^2$

f je INJ $\Leftrightarrow$ $\mathrm{Ker} f = \{0\}$

f je SUR $\Leftrightarrow$ $\mathrm{Im} f = \bar{V}$

4 11-18:18

4 11-18:22

4) $f((a,b,c)) = (a^2, -2b)$

$\mathbb{R}^3 \to \mathbb{R}^2$

$f((u_1,u_2,u_3) + (v_1,v_2,v_3)) = f((u_1,u_2,u_3)) + f((v_1,v_2,v_3))$

$L = f((u_1+v_1, u_2+v_2, u_3+v_3)) = ((u_1+v_1)^2, \underline{-2(u_2+v_2)})$

$P = (u_1^2, -2u_2) + (v_1^2, -2v_2)$

$= ((u_1^2+v_1^2), \underline{-2u_2-2v_2})$

$u_1 = 1$ $(u_1+v_1)^2 = 9$

$u_2 = 2$ $u_1^2+v_1^2 = 5$

4 11-18:13

$V=\mathbb{R}^3$

$V=\mathbb{R}^3$

$$f(x_1,x_2,x_3)=\begin{pmatrix}1&2&-1\\0&-1&2\\0&3&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$f(\underline{x})=A\cdot\underline{x}$

$f(\underline{x}+\underline{y})=f(\underline{x})+f(\underline{y})$

$A\cdot(\underline{x}+\underline{y})=A\cdot\underline{x}+A\cdot\underline{y}$

$f(\underline{x})=A\cdot\underline{x}$

$f(\underline{y})=A\cdot\underline{y}$

$f(t\underline{x})=t\,f(\underline{x})$

$f(t\cdot\underline{x})=A(t\underline{x})=t\cdot(A\cdot x)$

$t\cdot f(\underline{x})=t\cdot(A\cdot\underline{x})$ //

$$f(x_1,x_2,x_3)=\begin{pmatrix}1&2&-1\\0&-1&2\\0&3&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=$$

$=(x_1+2x_2-x_3,\ -x_2+2x_3,\ 3x+x_3)^T$

$f((a,b,c,d))=(a+b+c+d,\ -a-b-c-d,\ a-b+c-d,\ -2a+2b-2c+2d)$

$$f((a,b,c,d))=\begin{pmatrix}1&1&1&1\\-1&-1&-1&-1\\1&-1&1&-1\\-2&2&-2&2\end{pmatrix}\begin{pmatrix}a\\b\\c\\d\end{pmatrix}$$

4 11-18:25

$$f((a,b,c,d))=\begin{pmatrix}1&1&1&1\\-1&-1&-1&-1\\1&-1&1&-1\\-2&2&-2&2\end{pmatrix}\begin{pmatrix}a\\b\\c\\d\end{pmatrix}$$

$$f((1,0,0,0))=\begin{pmatrix}1&1&1&1\\-1&-1&-1&-1\\1&-1&1&-1\\-2&2&-2&2\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=$$

$(1,-1,1,-2)^T$

$f(0,1,0,0)=(1,-1,-1,2)^T$

$f(0,0,1,0)=(1,-1,1,-2)^T$

$f(0,0,0,1)=(1,-1,-1,2)^T$

$$\begin{pmatrix}1&-1&1&-2\\1&-1&-1&2\\1&-1&1&-2\\1&-1&-1&2\end{pmatrix}\sim\begin{pmatrix}1&-1&1&-2\\0&0&2&-4\\0&0&0&0\\0&0&2&-4\end{pmatrix}$$

$\mathrm{Im}\, f=\langle(1,-1,1,-2),(0,0,2,-4)\rangle$

$$\begin{pmatrix}1&1&1&1\\-1&-1&-1&-1\\1&-1&1&-1\\-2&2&-2&2\end{pmatrix}\sim\begin{pmatrix}1&1&1&1\\0&0&0&0\\0&2&0&2\\0&4&0&4\end{pmatrix}$$

$2b+2d=0$ $\quad d=t$

$b+d=0$ $\quad b=-t$

$b=-d$ $\quad c=s$

$\{(-s,-t,s,t)\mid s,t\in\mathbb{R}\}$ $\quad a=-s$

4 11-18:35

$\mathrm{Ker}\, f=\langle(1,0,0),(1,1,1)\rangle$

$\mathrm{Im}\, f=\langle 0,1,0\rangle$

$f((1,0,0))=(0,0,0)$

$f((1,1,1))=(0,0,0)$

$f(0,0,1)=(0,1,0)$

$u=t_1u_1+t_2u_2+t_3u_3$

$f(u)=t_1f(u_1)+t_2f(u_2)+t_3f(u_3)$

$(0,0,0)$ $\quad \begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\-1\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}$

$(0,1,0)=t_1(1,0,0)+t_2(1,1,1)+t_3(0,0,1)$

$t_1+t_2=0$ $\quad t_1=-1$

$t_2=1$

$t_2+t_3=0$ $\quad t_3=-1$

$f((0,1,0))=f(-1\cdot(1,0,0)+1\cdot(1,1,1)+(-1)t_3(0,0,1))$

$=(-1)\cdot f(1,0,0)+1\cdot f(1,1,1)+(-1)f(0,0,1)$

$=(0,0,0)+(0,0,0)-1\cdot(0,1,0)$

$=(0,-1,0)$

$\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\-1\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}$ $\quad f(x_1,x_2,x_3)=(0,x_3-x_2,0)$

$$f(x_1,x_2,x_3)=\begin{pmatrix}0&0&0\\0&-1&1\\0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$f(1,1,1)=\begin{pmatrix}0&0&0\\0&-1&1\\0&0&0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0\\1\\0\end{pmatrix}$$

4 11-18:43

$$f(x_1,x_2,x_3)=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$f(x_1,x_2,x_3)=\begin{pmatrix}a&0&0\\0&a&0\\0&0&a\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$\mathbb{R}^2$

$\sin 60=\frac{x}{1}$

$x=\frac{\sqrt{3}}{2}$

$y=\frac{1}{2}$

$(1,0)\mapsto\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

$(0,1)\mapsto\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$y=\frac{\sqrt{3}}{2}$

$x=-\frac{1}{2}$

$$f(x_1,x_2)=\begin{pmatrix}\frac{\sqrt{3}}{2}&-\frac{1}{2}\\\frac{1}{2}&\frac{\sqrt{3}}{2}\end{pmatrix}=\begin{pmatrix}\sin\varphi&-\cos\varphi\\\cos\varphi&\sin\varphi\end{pmatrix}$$

4 11-18:58