

Průběh Velet. pr. Zaměření

1)  $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i)  $B_1 \geq A$   $ax+by+cz+d$

ii)  $OR$

i) sázka podprostorů  
 $A_1: \exists A_1 \subset A_2$   
 $A_2: \exists B_1 \subset A_2$   
 $A_1 + A_2: Z(A_1) + Z(A_2) + AB$

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Průběh

Vzájemná poloha  $A_1, A_2$

$A_1 + A_2 = A_2$

$A_1 \subseteq A_2 \dots A_1 \cap A_2 = \emptyset$   $Z(A_1) \subseteq Z(A_2)$

$A_1 \not\subseteq A_2 \dots A_1 \cap A_2 \neq \emptyset$   $A_1 + A_2 \neq A_2$

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$A_1 \parallel A_2 \dots A_1 \cap A_2 = \emptyset$   $Z(A_1) \cap Z(A_2) = \emptyset$

$A_1 \not\parallel A_2 \dots A_1 \cap A_2 = \emptyset$   $Z(A_1) \cap Z(A_2) = \emptyset$

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i) ort. projekci  $u \rightarrow$  podprostor

ii) vzdálenost podprostorů

$u$  ortogon. proj  $AB$  do podpr.  $Z(a)^\perp$

$\|u\|$

ortog. projekci  $AB$  do podprostorů  $[Z(a) + Z(b)]^\perp$

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$A[4;7;-5]$   
 $B[2;-3;0]$

$P: 2x_1 - 2x_2 + x_3 - 9 = 0$   
 $Q: -x_1 + 2x_2 + x_3 - 12 = 0$

$2x_1 - x_3 - 9 = 0$   
 $x_3 = 2x_1 - 9$

$x_2 = 2x_1 - 9 + 9 = 2x_1$

$2x_1 = 8 - 2x_1 - 4x_1 - 9 + 2x_1 + 9$   
 $2x_1 = 6 - 2x_1$   
 $x_1 = 3$

$x_2 = 6$   
 $x_3 = 3$

$P: x_1 = 3$   
 $Q: x_2 = 6$   
 $x_3 = 3$

$Z(P) = \langle (-1, 1, 0, 2), (0, 1, 2, 0) \rangle$   
 $Z(Q) = \langle (1, 2, 1, -1) \rangle$

$-a+b+2d=0$   
 $b+2c+d=0$   
 $-a-2c+d=0$

$c=t$   
 $d=s$   
 $a=-2t+s$

$Z(P) = \langle (-2t+s, t, t, s) \rangle + tR^3$   
 $Z(Q) = \langle (-2, -2, 1, 0), (1, -1, 0, 0) \rangle$

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$Z(P) = \langle (-2, -2, 1, 0), (1, -1, 0, 0) \rangle$

$A[4;7;-5]$   
 $B[2;-3;0]$   
 $\vec{AB} = (-1, -3, 5, -1)$

$u = x + y$   
 $x = t(-2, -2, 1, 0) + s(1, -1, 0, 0)$

$u = t(-2, -2, 1, 0) + s(1, -1, 0, 0) + (0, 0, 0, 0)$

$(-1, -3, 5, -1) \cdot (-2, -2, 1, 0) = 2 + 6 + 5 = 13$   
 $(-1, -3, 5, -1) \cdot (1, -1, 0, 0) = -1 + 3 = 2$

$(-2, -2, 1, 0) \cdot (-2, -2, 1, 0) = 4 + 4 + 1 = 9$   
 $(1, -1, 0, 0) \cdot (1, -1, 0, 0) = 1 + 1 = 2$

$13 = 9t + 2s$   
 $2 = 2t + s$

$s = 2 - 2t$   
 $13 = 9t + 2(2 - 2t) = 9t + 4 - 4t = 5t + 4$   
 $9 = 5t$   
 $t = 1.8$   
 $s = 2 - 2(1.8) = 0.4$

$x = 1.8(-2, -2, 1, 0) + 0.4(1, -1, 0, 0)$

$\|x\| = \sqrt{x \cdot x}$

1)  $OR$   
 2)  $Z(A)$   
 3)  $Z(B)$   
 4)  $AB$   
 5) ort. pr.  $AB \rightarrow Z(A)$

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$P: [3; 5; 0] + t[1; -1; 0]$   
 $Q: x_1 - x_2 + 7 = 0$   
 $x_2 + 2x_3 - 5 = 0$   
 $x_1 - 5 = 0$

i)  $Q \perp P$   $Q \perp B_1, B_2$

ii)  $P \perp B_1, B_2$

iii)  $Z(Q) + Z(P) = W$

iv)  $W^\perp$

v)  $AB$

vi) ort. pr.  $AB \Rightarrow W^\perp \dots OR$

vii)  $\|u\|$

$Q: x_1 - x_2 + 7 = 0$   
 $x_2 + 2x_3 - 5 = 0$   
 $x_1 - 5 = 0$

$x_1 = 5$   
 $x_2 = 5 - 2x_3$   
 $x_3 = t$   
 $x_2 = 5 - 2t$   
 $x_1 = 5$

$P: [5; 5 - 2t; t]$   
 $Q: [5; 5 - 2t; t]$

$Z(P) = \langle (2, 1, 2), (1, -2, 1) \rangle$

$W^\perp = \langle (4, 1, 1) \rangle$

$2a - 3c + d = 0$   
 $2a - 2b + c = 0$   
 $-2b - 5c - 2d = 0$

$c = t$   
 $d = 2t$   
 $a = 2t - 2t = 0$   
 $b = 2t - 2t = 0$

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$W = \langle (3s, 2t, 0), (1, 2, 0-2) \rangle$   
 $W^\perp = \langle (1, 0, 1), 2 \rangle$   
 $M = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}$   
 $P = \begin{bmatrix} -7 & 5 & 0 & 5 \\ 7 & 5 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} 1 & -2 & 1 & 0 \end{bmatrix}$   
 $P = \begin{bmatrix} 7 & 5 & 0 & 1 \end{bmatrix} + s \begin{bmatrix} 3 & 0 & 3 & 1 \end{bmatrix}$   
 $M = x + y$   
 $M = t(3s, 2t, 0) + s(1, 2, 0-2)$   
 $M = t(3s, 2t, 0) + s(1, 2, 0-2)$   
 $(1, 0, 1) \cdot (3s, 2t, 0) = 2t$   
 $(1, 0, 1) \cdot (1, 2, 0-2) = 18$   
 $(3s, 2t, 0) \cdot (1, 2, 0-2) = 13$   
 $(3s, 2t, 0) \cdot (3s, 2t, 0) = 38$   
 $(1, 2, 0-2) \cdot (1, 2, 0-2) = 9$   
 $2t = 38t + 13s$   
 $18 = 13t + 9s$

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$P = A t -$   
 $r = B t -$   
 1)  $r$ : Param.  
 2)  $2(p) + 2(r) = W$   
 3)  $W^\perp$   
 4)  $\overrightarrow{AB}$   
 5) OP  $\overrightarrow{AB} \rightarrow W^\perp = x$   
 6)  $\|x\|$

5 2-18:55

$\cos \varphi = \frac{|u \cdot v|}{\|u\| \|v\|}$   
 a)  $P$  podprůměr  
 $30^\circ - \varphi$   
 b)  $\varphi$   
 $f(u, v) = f(v, u)$   
 c)  $P$  podprůměr  
 $P, B, z(P)$   
 $Q, A, z(Q)$   
 $z(P) \cap z(Q) = z$   
 $z(P) \cap z(Q) = z$   
 $f(u, v) = f(v, u)$

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$M = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$   
 $B: x + y + z = 7$   
 $\cos \varphi = \frac{|u \cdot v|}{\|u\| \|v\|}$   
 $M = (2, 1, 1) \cdot (1, 1, 1) = 3$   
 $\|u\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} = 3$   
 $\|v\| = \sqrt{1^2 + 1^2 + 1^2} = 2$   
 $\cos \varphi = \frac{3}{6} = \frac{1}{2}$   
 $\varphi = \frac{\pi}{3}$   
 $f(P, P) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$   
 $M = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$   
 $P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + t \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$   
 $z(P) = \langle (2, 1, 1), (1, 1, 1) \rangle$   
 $M = x + y$   
 $x = t(2, 1, 1) + s(1, 1, 1)$   
 $M = t(2, 1, 1) + s(1, 1, 1) + y$   
 $M_{11} = (2, 1, 1) \cdot (2, 1, 1) = 10$   
 $M_{12} = (2, 1, 1) \cdot (1, 1, 1) = 5$   
 $M_{21} = (1, 1, 1) \cdot (2, 1, 1) = 5$   
 $M_{22} = (1, 1, 1) \cdot (1, 1, 1) = 10$   
 $M_{33} = 5$   
 $10 = 5t + 5s$   
 $5 = 5t + 5s$   
 $5 = 5s$   
 $s = 1 \Rightarrow t = 1$

5 2-19:06

$U = \langle (2, 1, 1), (1, 1, 1) \rangle$   
 $V = \langle (1, 1, 1), (1, 1, 1) \rangle$   
 i)  $z(U) \cap z(V) = W$   
 ii)  $W^\perp$   
 iii)  $U \cap V = W$   
 iv)  $V \cap W = W$   
 v)  $U \cap W = W$   
 $U \cap V = \langle (1, 1, 1) \rangle$   
 $U \cap W = \langle (1, 1, 1) \rangle$   
 $V \cap W = \langle (1, 1, 1) \rangle$   
 $U \cap V \cap W = \langle (1, 1, 1) \rangle$   
 $U \cap V = \langle (1, 1, 1) \rangle$   
 $U \cap W = \langle (1, 1, 1) \rangle$   
 $V \cap W = \langle (1, 1, 1) \rangle$   
 $U \cap V \cap W = \langle (1, 1, 1) \rangle$   
 $U \cap V = \langle (1, 1, 1) \rangle$   
 $V \cap W = \langle (1, 1, 1) \rangle$   
 $U \cap V \cap W = \langle (1, 1, 1) \rangle$

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$r + q = 0$   
 $W = \langle (p+q, r+p+q, p+q, p+q, r+q) \rangle$   
 $W = \langle (p+q, p, p+q, p+q, 0) \rangle$

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