M8101 - zk3 - 8A

(1) $p: [1,-2,0] + t\underbrace{(2,3,1)}_{u_1}$; $t \in \mathbb{R}$

$q: \begin{matrix} x+y-z=1 \\ -3x+y+z=9 \end{matrix}$ param. tvar např. $[0,5,4] + s\overbrace{(2,2,4)}^{u_2}$

Vektor ortog. k $(2,3,1)$ i $(2,2,4)$ ... $(5,-3,-1) = v$

Rovina p, v : $[-1,-2,0] + t(2,3,1) + r(5,-3,-1)$

Průsečík s q: $[-1,4,2]$, tj. příčka má rovnici $\underline{[-1,4,2] + r(5,-3,-1)}$

její průsečík s p: $\left[\frac{23}{7}, \frac{10}{7}, \frac{8}{7}\right]$.

Vzdálenost mimoběžek $d^2 = \left(\frac{30}{7}\right)^2 + \left(\frac{18}{7}\right)^2 + \left(\frac{6}{7}\right)^2 = 35 \cdot \left(\frac{6}{7}\right)^2 \Rightarrow d = \frac{6}{7}\sqrt{35}$

<u>Alt.</u> $d = \frac{|(B-A)\cdot(u_1 \times u_2)|}{|u_1 \times u_2|} = \frac{6}{7}\sqrt{35}$

(2) $\begin{vmatrix} 6-\lambda & 9 & -4 \\ -5 & -7-\lambda & 3 \\ -4 & -5 & 2-\lambda \end{vmatrix} = (6-\lambda)(7+\lambda)(\lambda-2) + 27\cdot(-4) + 25\cdot(-4) + 16(7+\lambda) + 15(6-\lambda) + 45(2-\lambda)$

$= -\lambda^3 + \lambda^2(2+6-7) + \lambda(14 - 12 + \overset{-84}{42}) + 52\cdot(-4) + 112 + 90 + 90 - 44\lambda =$

$= -\lambda^3 + \lambda^2 = \lambda^2(-\lambda+1)$

$= 0 \Leftrightarrow \lambda_{1,2} = 0$ alg. nás. 2

$\underline{\lambda_3 = 1}$ alg. nás. 1

$\lambda = 0$: $\begin{pmatrix} 6 & 9 & -4 \\ -5 & -7 & 3 \\ -4 & -5 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 4 & -2 \\ -1 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 0 & 3 & -2 \end{pmatrix}$

Vodíme $u_3 = 3$
$3u_2 - 2u_3 = 0 \Rightarrow u_2 = 2$
$u_1 + 2u_2 - u_3 = 0 \Rightarrow u_1 + 4 - 3 = 0$

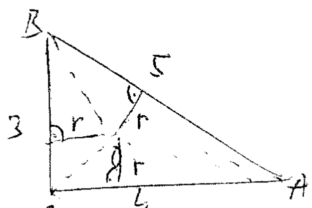
$u = (-1, 2, 3)$ geom. nás. $\lambda = 0$ je 1.

$\lambda = 1$: $\begin{pmatrix} 5 & 9 & -4 \\ -5 & -8 & 3 \\ -4 & -5 & 1 \end{pmatrix} \sim \begin{pmatrix} 5 & 9 & -4 \\ 0 & 1 & -1 \\ -4 & -5 & 1 \end{pmatrix} \sim \begin{pmatrix} 5 & 9 & -4 \\ 0 & 1 & -1 \\ -4 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} 5 & 9 & -4 \\ 0 & 1 & -1 \\ +1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 9 & -9 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \sim$

$\sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ $\begin{matrix} u_3 = 1 \\ u_2 = 1 \\ u_1 = -1 \end{matrix}$ $u = (-1, 1, 1)$ geom. nás. je 1 (už jsme věděli bez výpočtu 4)

Matice M <u>není</u> diagonalizovatelná (součet geom. nás. je $2 < 3$)

(3)



a) střed kružnice vepsané (tj. průsečík os vnitřních úhlů)

b) $P = 1 - \frac{S_{\Delta DAB}}{S_{\Delta ABC}} = 1 - \frac{\frac{1}{2}5\cdot r}{6r} = \frac{7}{12}$

r .. poloměr kr. vepsané

$S_{\Delta ABC} = \frac{1}{2}(5r + 4r + 3r) = 6r \wedge S_{\Delta ABC} = \frac{3\cdot 4}{2} = 6 \Rightarrow r = 1$ (Výpočet r nebyl třeba!)

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MB101-383

(4) $u_1 = (1,t,2,0)$, $u_2 = (-1,1,0,0)$, $u_3 = (1,-2,2,3)$, $u_4 = (2,-5,6,3)$.

a) $\left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 1 & -2 & -5 & t \\ 0 & 2 & 6 & 2 \\ 0 & 3 & 3 & 0 \end{array}\right) \sim \dots \sim \left(\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & t+2 \\ 0 & 0 & 2 & 1 \end{array}\right)$.

$u_1 \in \langle u_2, u_3, u_4 \rangle \iff t = -2$

b)

$v_2 = u_2$

$v_3 = u_3 + p \cdot v_2$; $p = \frac{3}{2}$

$v_3 = (-\frac{1}{2}, -\frac{1}{2}, 2, 3)$

$v_4 = u_4 + p_1 \cdot v_2 + p_2 \cdot v_3$; $p_1 = -\frac{v_2 \cdot u_4}{v_2 \cdot v_2} = \frac{7}{2}$; $p_2 = -\frac{v_3 \cdot u_4}{v_3 \cdot v_3} = -\frac{5}{3}$

$v_4 = (-\frac{2}{3}, -\frac{2}{3}, \frac{8}{3}, -2)$

ortog. báze je např. $\alpha = (-1,1,0,0), (-1,-1,4,6), (-1,-1,4,-3)$

c) $\left(\begin{array}{ccc|c} -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -2 \\ 0 & 4 & 4 & 2 \\ 0 & 6 & -3 & 0 \end{array}\right) \sim \dots \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $\Rightarrow$ souřadnice u_1 v bázi α jsou $[-\frac{3}{2}, \frac{1}{6}, \frac{1}{3}]$

(5) $A = \{1,2,3,4,5\}$, $B = \{a,b,c,d\}$

$\rho = \{[1,c], [2,b], [3,a], [4,d], [5,b]\}$

a) je zobrazení, surjektivní, není injektivní

b) $\rho^{-1} = \{[c,1], [b,2], [a,3], [d,4], [b,5]\}$

není zobrazení ($b \nearrow 2$, $b \searrow 5$)

c) $\rho^{-1} \circ \rho = \{[1,1], [2,2], [2,5], [3,3], [4,4], [5,2], [5,5]\}$

$\rho \circ \rho^{-1} = \{[c,c], [b,b], [a,a], [d,d]\}$