I

$$\int \sqrt[3]{\frac{x-1}{x+1}}\cdot\frac{1}{x-1}\,dx =$$

$$\frac{x-1}{x+1}=t^3$$
$$x-1=t^3(x+1)$$
$$x-t^3x=t^3+1$$
$$x(1-t^3)=t^3+1$$
$$x=\frac{t^3+1}{1-t^3}$$
$$dx=\frac{3t^2(1-t^3)+(t^3+1)\cdot 3t^2}{(1-t^3)^2}\,dt=\frac{3t^2(1-t^3+t^3+1)}{(1-t^3)^2}=\frac{6t^2}{(1-t^3)^2}\,dt$$

$$=\int\sqrt[3]{t^3}\cdot\frac{1}{\frac{t^3+1}{1-t^3}-1}\cdot\frac{6t^2}{(1-t^3)^2}\,dt$$

$$=\int t\cdot\frac{1}{\frac{t^3+1-1+t^3}{1-t^3}}\cdot\frac{6t^2\,dt}{(1-t^3)^2}=$$

$$=\int\frac{\cancel{6}^{3}\cancel{t^3}\cdot\cancel{(1-t^3)}}{(1-t^3)^{\cancel{2}}\cdot\cancel{2t^3}}\,dt=3\int\frac{1}{1-t^3}\,dt=$$

$$=3\int\frac{1}{(1-t)(1+t+t^2)}\,dt=3\int\left(\frac{A}{1-t}+\frac{Bt+C}{1+t+t^2}\right)dt=$$

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$$A(1+t+t^2)+(Bt+C)(1-t)=1$$

$t=1 \Rightarrow \quad 3A=1 \Rightarrow \underline{\underline{A=1/3}}$

$t^2: \Rightarrow \quad A-B=0 \Rightarrow \underline{\underline{B=1/3}}$

$t: \Rightarrow \quad A+B-C=0 \Rightarrow \underline{\underline{C=2/3}}$

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$$=\cancel{3}\int\frac{1}{\cancel{3}(1-t)}\,dt+\cancel{3}\int\frac{t+2}{\cancel{3}(1+t+t^2)}\,dt=\int\frac{-1}{-1+t}\,dt+\int\frac{t+2}{1+t+t^2}\,dt=$$

$$= \underline{\underline{-\ln|t-1|}} + \underbrace{\int\frac{t+2}{1+t+t^2}\,dt}_{(*)}$$

(1) →

$$(*)=\int\frac{t+2}{1+t+t^2}\,dt=\int\frac{2t+4}{2(1+t+t^2)}\,dt=\frac{1}{2}\int\frac{2t+1+3}{1+t+t^2}\,dt=$$

$$=\frac{1}{2}\left[\int\frac{2t+1}{1+t+t^2}\,dt+3\int\frac{1}{1+t+t^2}\,dt\right]=\underline{\underline{\frac{1}{2}\ln|1+t+t^2|}}+\underbrace{\frac{3}{2}\int\frac{1}{1+t+t^2}\,dt}_{(+)}$$

(2)

=> DRUHÝ LIST

II

$$(+) = \frac{3}{2}\int \frac{1}{t^2+t+1}\,dt = \frac{3}{2}\int \frac{1}{(t+1/2)^2+\frac{3}{4}}\,dt =$$

$$= \frac{3}{2}\int \frac{1}{\frac{3}{4}\left[\left(\frac{t+1/2}{\frac{\sqrt{3}}{2}}\right)^2+1\right]}\,dt =$$

vypadá jako $\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$ → + úprava

$$= \frac{3}{2}\cdot\frac{4}{3}\int \frac{1}{\left(\frac{2t+1}{\sqrt{3}}\right)^2+1}\,dt = 2\cdot\int \frac{1}{\left(\frac{2t+1}{\sqrt{3}}\right)^2+1}\,dt =$$

$$= 2\cdot\frac{\sqrt{3}}{2}\cdot\operatorname{arctg}\left(\frac{2t+1}{\sqrt{3}}\right) = \sqrt{3}\operatorname{arctg}\left(\frac{2t+1}{\sqrt{3}}\right)$$

(3)

Tedy celkově:

$$(1)+(2)+(3) = -\ln|t-1| + \frac{1}{2}\ln|1+t+t^2| + \sqrt{3}\operatorname{arctg}\left(\frac{2t+1}{\sqrt{3}}\right) + C$$

Využije se tabulkového $\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$, s úpravou: $\frac{1}{a}\cdot\operatorname{arctg}\left(\frac{x-x_0}{a}\right)$, kde $a$ je zřejmě $\frac{\sqrt{3}}{2}$,

což když se dá samostatně (zakroužkované červeně)

$$\frac{4}{3}\cdot\frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}} = \frac{1}{a}$$

Nakonec DOSAĎTE ZA $t$ !!!