- $\int(1-\sqrt{x})^2\,dx = \int(1-2\sqrt{x}+x)\,dx = x-2\,\frac{x^{3/2}}{3/2}+\frac{x^2}{2}+C$

- $\int tg^2x\,dx = \int\frac{\sin^2x}{\cos^2x}\,dx = \int\frac{1-\cos^2x}{\cos^2x}\,dx = \int\left(\frac{1}{\cos^2x}-1\right)dx = tg\,x - x + C$

- $\int cotg\,x\,dx = \int\frac{\cos x}{\sin x}\,dx = \int\frac{(\sin x)'}{\sin x}\,dx = \ln|\sin x| + C$

$\cos^2x+\sin^2x=1$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$\int\left(\sqrt{x^3}-\frac{1}{\sqrt{x}}\right)dx = \frac{x^{5/2}}{5/2}-\frac{\sqrt{x}}{1/2}+C$

- $\int\frac{x^3-1}{x-1}\,dx = \int\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}\,dx = \frac{1}{3}x^3+\frac{1}{2}x^2+x+C$

- $\int\frac{(2^x-3^x)^2}{6^x}\,dx = \int 6^{-x}(2^x-3^x)^2\,dx = \int 6^{-x}(4^x-2\cdot6^x+9^x)\,dx =$

$= \int\left[\left(\frac{2}{3}\right)^x - 2\cdot1^x + \left(\frac{3}{2}\right)^x\right]dx = \frac{\left(\frac{2}{3}\right)^x}{\ln\frac{2}{3}} - 2x + \frac{\left(\frac{3}{2}\right)^x}{\ln\frac{3}{2}}$

$\int cotg^2x\,dx = \int\frac{\cos^2x}{\sin^2x}\,dx = \int\frac{1-\sin^2x}{\sin^2x}\,dx = -cotg\,x - x + C$

- $\int(5^x+2\cdot3^{-x}-4\cos x)\,dx = \frac{5^x}{\ln5}-2\frac{3^{-x}}{\ln3}-4\sin x+C$

PER PARTES:

$\int x\ln x\,dx = \begin{vmatrix} u'=x & v=\ln x \\ u=\frac{x^2}{2} & v'=\frac{1}{x}\end{vmatrix} = \frac{x^2}{2}\ln x - \int\frac{x^2}{2}\cdot\frac{1}{\cancel{x}}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}+C$ na $(0,\infty)$

$\int e^x(x^3-2x)\,dx = \begin{vmatrix} u'=e^x & v=x^3-2x \\ u=e^x & v'=3x^2-2\end{vmatrix} = e^x(x^3-2x)-\int e^x(3x^2-2)\,dx = \begin{vmatrix} u'=e^x & v=3x^2-2 \\ u=e^x & v'=6x\end{vmatrix} =$

$= e^x(x^3-2x)-\left[e^x(3x^2-2)-\int e^x\cdot6x\,dx\right] = \begin{vmatrix} u'=e^x & v=6x \\ u=e^x & v'=6\end{vmatrix} = \cancel{e^x6x-\int e^x\cdot6\,dx =}$

$\cancel{= e^x6x-6e^x+C}$

$= e^x(x^3-2x)-e^x(3x^2-2)+\left[e^x6x-\int e^x\cdot6\,dx\right] =$

$= \text{——— " ———} - 6e^x + C$

$= e^x(x^3-3x^2+4x-4)+C$

Substituce

- $\int \sin x \cos x \, dx = \left| \begin{array}{l} y = \cos x \\ dy = -\sin x \, dx \\ dx = \frac{dy}{-\sin x} \end{array} \right| = \int \cancel{\sin x} \cdot y \cdot \frac{dy}{-\cancel{\sin x}} = -\int y \, dy = -\frac{y^2}{2} + C =$

  $2\sin x \cos x = \sin 2x$

  $= -\frac{\cos^2 x}{2} + C$

- $\int \frac{dx}{x^2+9} = \frac{1}{9}\int \frac{dx}{\left(\frac{x}{3}\right)^2+1} = \left| \begin{array}{l} \frac{x}{3} = y \\ dy = \frac{1}{3}dx \end{array} \right| = \frac{1}{3}\int \frac{dy}{y^2+1} = \frac{1}{3}\operatorname{arctg} y + c = \frac{1}{3}\operatorname{arctg}\frac{x}{3} + C$

- $\int \frac{dx}{x \ln x} = \left| \begin{array}{l} y = \ln x \\ dy = \frac{1}{x}dx \end{array} \right| = \int \frac{\cancel{x}\, dy}{\cancel{x}\, y} = \ln|y| + c = \ln|\ln(x)| + c$

- $\int \frac{1}{\sin^2(-5x)} dx = \left| \begin{array}{l} y = -5x \\ dy = -5dx \\ dx = -\frac{dy}{5} \end{array} \right| = \int -\frac{1}{5}\frac{dy}{\sin^2 y} = +\frac{1}{5}\operatorname{cotg} y + c = \frac{1}{5}\operatorname{cotg}(-5x) + c$

- $\int \frac{2x}{x^2-1} dx = \left| \begin{array}{l} y = x^2-1 \\ dy = 2x\,dx \end{array} \right| = \int \frac{dy}{y} = \ln|u| + c = \ln|x^2-1| + c$

  zkouškový

  lze řešit také rozkladem na parciální zlomky, nebo vydělením polyn.

  vzorec $\int \frac{f'(x)}{f(x)}$!

[redacted]

PER PARTES

- $\int x \cdot \ln^2 x \, dx = \left| \begin{matrix} u' = x & v = \ln^2 x \\ u = \frac{x^2}{2} & v' = 2\ln x \cdot \frac{1}{x} \end{matrix} \right| = \frac{x^2}{2}\ln^2 x - \int \frac{x^2}{2} \cdot 2\ln x \cdot \frac{1}{x} \, dx = \left| \begin{matrix} u' = x & v = \ln x \\ u = \frac{x^2}{2} & v' = \frac{1}{x} \end{matrix} \right|$

zkouškový

$= \frac{x^2}{2}\ln^2 x - \left[\frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx \right] = \frac{x^2}{2}\ln^2 x - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$

$\int (x^2+5)\cos 2x \, dx = \left| \begin{matrix} u' = \cos 2x & v = x^2+5 \\ u = \frac{1}{2}\sin 2x & v' = 2x \end{matrix} \right| = \ldots 2x$ per partes $\ldots = \left(\frac{x^2}{2} + \frac{9}{4}\right)\sin 2x + \frac{x}{2}\cos 2x + C$

(der. polynom)

$\int (5x^4+6x^2-6x)\ln 4x \, dx = \left| \begin{matrix} u' = 5x^4+6x^2-6x & v = \ln 4x \\ u = x^5 + 6\frac{x^3}{3} - 3x^2 & v' = \frac{1}{x} \end{matrix} \right| = \ldots\ldots =$

$= (x^5+2x^3-3x^2)\ln 4x - \frac{x^5}{5} - \frac{2x^3}{3} + \frac{3x^2}{2} + C$