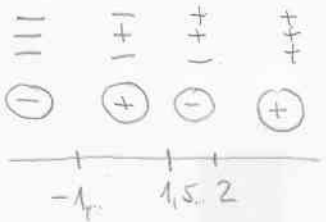
Př. 1 (7,5 bodu)

- $f(x) = \frac{2x^2 - x - 4}{x-2} \rightarrow x \neq 2 \rightarrow \mathcal{D}(f) = \mathbb{R} \setminus \{2\}$

$2x^2 - x - 4 \rightarrow x_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 4 \cdot 2}}{4} = \frac{1 \pm \sqrt{33}}{4} = \begin{cases} -1,\dots \\ 1,5\dots \end{cases}$ ($5 < \sqrt{33} < 6$)

stačil by mi desetiny tvar



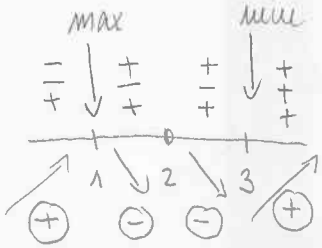
$\Rightarrow f(x) > 0 \dots x \in \left(\frac{1-\sqrt{33}}{4}, \frac{1+\sqrt{33}}{4}\right) \cup (2, \infty)$

$f(x) < 0 \dots x \in \left(-\infty, \frac{1-\sqrt{33}}{4}\right) \cup \left(\frac{1+\sqrt{33}}{4}, 2\right)$

1b

- $f'(x) = \frac{2x^2 - 8x + 6}{(x-2)^2} \rightarrow x \neq 2$

$\hookrightarrow 2x^2 - 8x + 6 \rightarrow x_{1,2} = \frac{8 \pm \sqrt{64 - 4 \cdot 6 \cdot 2}}{4} = \frac{8 \pm 4}{4} = \begin{cases} 3 \\ 1 \end{cases}$



$\Rightarrow f$ rostoucí $\dots x \in (-\infty, 1] \cup [3, \infty)$

f klesající $\dots x \in [1, 2) \cup (2, 3]$

$\rightarrow$ max.: $x = 1 \rightarrow f(1) = \frac{2 - 1 - 4}{1-2} = 3$

min.: $x = 3 \rightarrow f(3) = \frac{2 \cdot 9 - 3 - 4}{3-2} = 11$

2b

- $f''(x) = \frac{\overbrace{2x^2 - 8x + 10}^{\text{vždy } (+)}}{(x-2)^3}$

$\hookrightarrow 2x^2 - 8x + 10 \rightarrow x_{1,2} = \frac{8 \pm \sqrt{\overbrace{64 - 4 \cdot 2 \cdot 10}^{<0}}}{4}$

$\Rightarrow f$ konvexní $\dots x \in (2, \infty)$

f konkávní $\dots x \in (-\infty, 2)$

2b

inflexní bod není

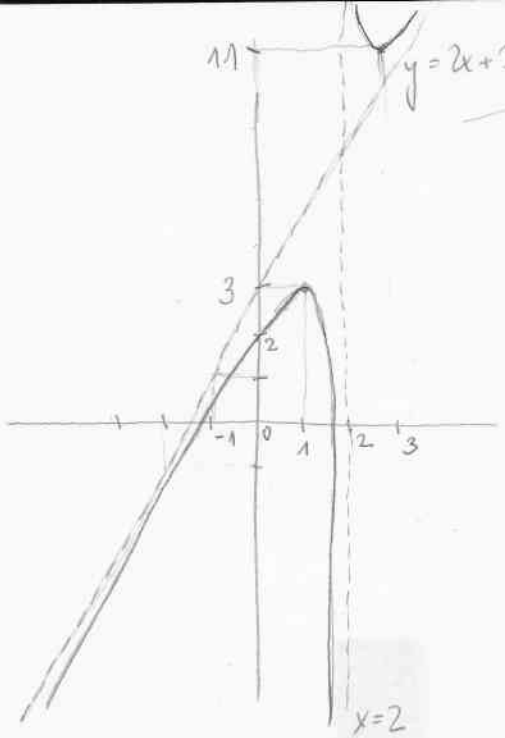
asympt. bez sm.: 1,5b

$x = 2: \lim_{x \to 2} f(x) = \frac{\sim}{0} = \infty = \boxed{x = 2}$

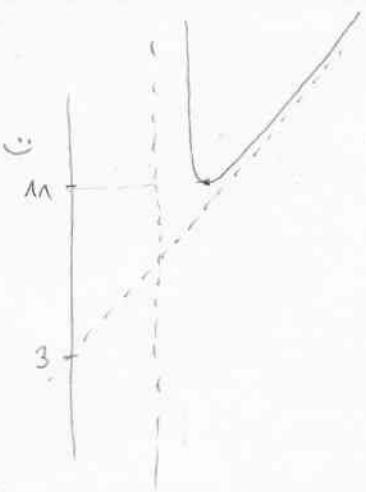
asympt. se sm.: $y = ax + b$

$a = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \frac{2x^2 - x - 4}{x^2 - 2x} = 2$, $b = \lim_{x \to \pm\infty} (f(x) - ax) = \lim_{x \to \pm\infty} \frac{2x^2 - x - 4 - 2x^2 + 4x}{x-2} = \overbrace{\lim_{x \to \pm\infty} \frac{3x - 4}{x - 2}}^{=3}$

$\Rightarrow \boxed{y = 2x + 3}$

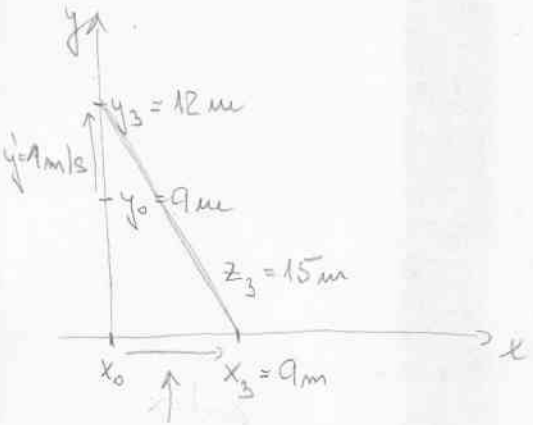


nevešlo se mi to tam ☺



1b

Př. 2 (7,5 b)



y... výška balónu v čase t

x... vzdálenost chodce na ceste v čase t

z... vzdálenost chodce a balónu v čase t

$x_0 = 0$, $y_0 = 9$ } za 3 s: $x_3 = 0 + 3 \cdot 3 = 9$ m

$y_3 = 9 + 3 \cdot 1 = 12$ m

$z^2 = x^2 + y^2 = 9^2 + 12^2 = \sqrt{81 + 144} = \sqrt{225} = 15$ m

$\rightarrow 2z \cdot z' = 2x x' + 2y y' \Rightarrow z' = \frac{x x' + y y'}{z} = \frac{9 \cdot 3 + 12 \cdot 1}{15} = \frac{39}{15} = 2{,}6$ m/s

⇒ Budou se od sebe po 3 sekundě vzdalovat rychlostí 2,6 m/s.