Př. 3 (5 bodů)

$$y' = \frac{x(y+1)}{x+1}$$

$$\rightarrow \int \frac{dy}{y+1} = \int \frac{x}{x+1}\,dx \quad , \; y \neq -1$$

$$\int \frac{dy}{y+1} = \ln|y+1|$$

$$\int \frac{x}{x+1}\,dx = \int \left(1 - \frac{1}{x+1}\right) dx = x - \ln|x+1| + C$$

nebo: $C = \ln|K|$, $K \neq 0$

$$\Rightarrow y = \frac{K}{x+1} e^{x} - 1 , \quad K \in \mathbb{R}$$

$$\Rightarrow \ln|y+1| = x - \ln|x+1| + C$$

$$y+1 = e^{x - \ln|x+1| + C} = e^{x+C} \cdot \frac{1}{x+1}$$

$$y = \frac{e^{x+C}}{x+1} - 1 , \quad C \in \mathbb{R}$$

$$y = -1$$

poč. podm.: $y(0) = e-1$:

$$e-1 = \frac{e^{0+C}}{0+1} - 1 = e^{C} - 1 \Rightarrow C = 1$$

$K = e$

$$\Rightarrow y = \frac{e^{x+1}}{x+1} - 1$$

Př. 1 (5 bodů)

$$\sum_{n=0}^{\infty} \frac{3^n}{n^3}$$

$\to$ podílové kritérium: $\lim\limits_{n\to\infty} \dfrac{\dfrac{3^{n+1}}{(n+1)^3}}{\dfrac{3^n}{n^3}} = \lim\limits_{n\to\infty} \dfrac{3n^3}{(n+1)^3} = 3 > 1$

$\Rightarrow$ řada diverguje

Př. 2 (5 bodů)

$$\sum_{n=0}^{\infty} (n+2)x^n$$

$a = \lim\limits_{n\to\infty} \dfrac{n+3}{n+2} = 1 \Rightarrow R = \dfrac{1}{a} = 1$ ... poloměr konvergence

$x=-1: \sum\limits_{n=0}^{\infty} (-1)^n (n+2)$

$\lim\limits_{n\to\infty} (n+2) = \infty \to$ diverguje

$x=1: \sum\limits_{n=0}^{\infty} (n+2) = \infty \to$ diverguje

$\Rightarrow$ obor konvergence: $x \in (-1,1)$

$$\sum_{n=0}^{\infty}(n+2)x^n = \sum_{n=0}^{\infty}(n+2)x^{n+1}\cdot x^{-1} = \sum_{n=0}^{\infty}\left(x^{n+2}\right)'\cdot x^{-1} = x^{-1}\Big[\overbrace{\sum_{n=0}^{\infty} x^{n+2}}^{x^2+x^3+x^4+\dots}\Big]' = x^{-1}\left(\frac{x^2}{1-x}\right)' =$$

$$= x^{-1}\,\frac{2x(1-x)+x^2}{(1-x)^2} = \frac{2(1-x)+x}{(1-x)^2} = \frac{2-x}{(1-x)^2}$$