A... druhá vytažená koule bude černá

B... první vytažená koule byla černá

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{5} = \frac{1}{6} + \frac{1}{10} = \frac{5+3}{30} = \frac{8}{30} = \frac{4}{15}$$

$$P(A \cap B) = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{5} + \frac{1}{2}\cdot\frac{1}{5}\cdot\frac{3}{4} = \frac{1}{15} + \frac{3}{40} = \frac{8+9}{120} = \frac{17}{120}$$

$$P(A/B) = \frac{\frac{17}{120}}{\frac{4}{15}} = \boxed{\frac{17}{32}}$$

2) A… 1. vytažená koule byla červ.

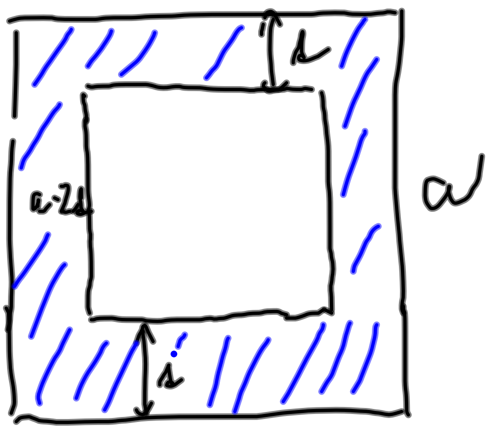
B… 3. vytažená koule byla bílá

$$P(B) = \frac{1}{2}\left(\underset{\bar{č}}{\frac{1}{3}} \cdot \underset{\bar{č}}{\frac{1}{5}} \cdot \underset{B}{\frac{1}{2}} + \underset{\bar{č}}{\frac{1}{3}} \cdot \underset{B}{\frac{2}{5}} \cdot \underset{B}{\frac{1}{4}} + \dots \right) + \frac{1}{2}\left(\dots \right)$$

$$= \frac{4}{15}$$

$$P(A \cap B) = \frac{17}{120}$$

$$P(A|B) = \frac{17}{32}$$



$$p = \frac{S}{S} = \frac{a^2-(a-2s)^2}{a^2} = \frac{4s(a-s)}{a^2}$$

$$P(\underbrace{\text{padne více hlav}}_{A}) = P(\text{padne více orlů}) =$$

$$= P(\underbrace{\text{padne nejvýše tolik hlav co orlů}}_{\bar{A}})$$

$$P(A) = P(\bar{A}) = 1 - P(A) \Rightarrow P(A) = \frac{1}{2}$$

Náhodná veličina ... číselný výsledek nějakého pokusu

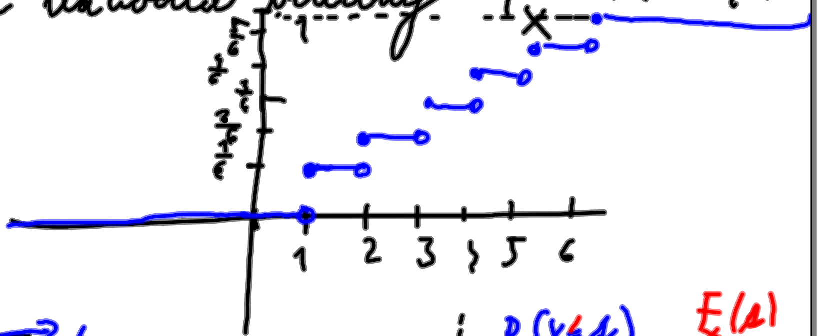
X ... číslo, které padne při hodu 1 kostkou

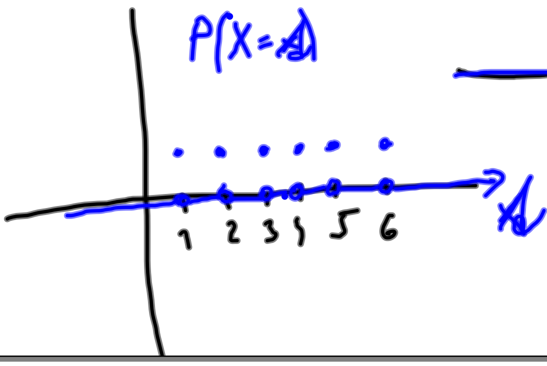
$P(X=1) = \frac{1}{6}$

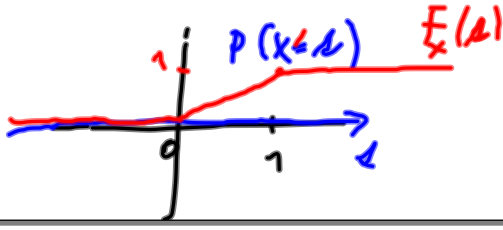
$P(X=i) = \frac{1}{6}$, $i \in \{1,2,3,4,5,6\}$

Distribuční funkce náhodné veličiny $F_X : R \to \langle 0,1 \rangle$

$F_X(s) := P(X \le s)$

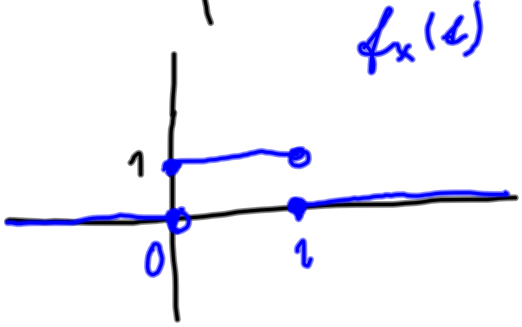






$F'(t) = f(t)$ ... hustota pravděpodobnosti

$P(X \in \langle t, t+\Delta t \rangle) = F(t+\Delta t) - F(t)$



$P(a \leq X \leq b) = \int_a^b f_X(t)\,dt$

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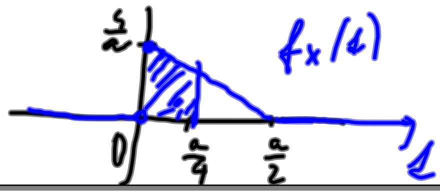
Ve čtverci o straně a zvolíme náhodný bod. Určete distr. funkci a hustotu pstí náhodné veličiny udávající vzdálenost tohoto bodu od nejbližší strany čtverce.

$0 \le X \le \frac{a}{2}$

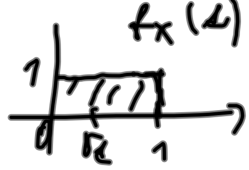
$$F_X(t) = \begin{cases} P(X<t) = \frac{4(a-t)t}{a^2}, & \text{pro } 0 \le t \le \frac{a}{2} \\ 0, & \text{pro } t<0 \\ 1, & \text{pro } t > \frac{a}{2} \end{cases}$$

$$(F_X(t))' = \left(\frac{4at - 4t^2}{a^2}\right)' = \frac{4}{a} - \frac{8t}{a^2}$$

$$f_X(t) = \begin{cases} 0, & \text{pro } t<0 \\ \frac{4}{a} - \frac{8t}{a^2} & \text{pro } 0 \le t < \frac{a}{2} \\ 0, & \text{pro } t \ge \frac{a}{2} \end{cases}$$



Y ... náhodná veličina popisující obsah čtverce se stranou X

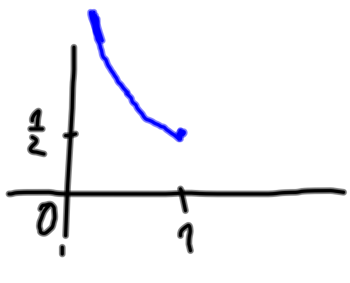


$0 \leq Y < 1$.

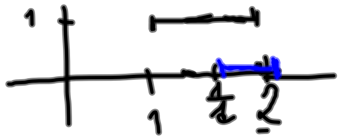
$$F_Y(t) = P(Y \leq t) = \qquad 0 < t < 1$$
$$= P(X^2 \leq t) = P(X \leq \sqrt{t}) = \sqrt{t}$$

$$F_Y(t) = \begin{cases} 0 & \text{pro } t \leq 0 \\ \sqrt{t} & \text{pro } 0 < t < 1 \\ 1 & \text{pro } t \geq 1 \end{cases}$$

$$f_Y(t) = \begin{cases} 0 & \text{pro } t \leq 0 \\ \frac{1}{2\sqrt{t}} & \text{pro } 0 < t < 1 \\ 0 & \text{pro } t \geq 1 \end{cases}$$



$Y := \frac{1}{X}$. $\quad Y \in \left(\frac{1}{2}, 1\right)$



$$F_Y(t) = \begin{cases} 0 & \text{pro } t \le \frac{1}{2} \\ P(Y \le t) = P\left(\frac{1}{X} < t\right) = P\left(X > \frac{1}{t}\right) = 2 - \frac{1}{t}. & \text{pro } \frac{1}{2} < t < 1 \\ 1 & \text{pro } t \ge 1 \end{cases}$$

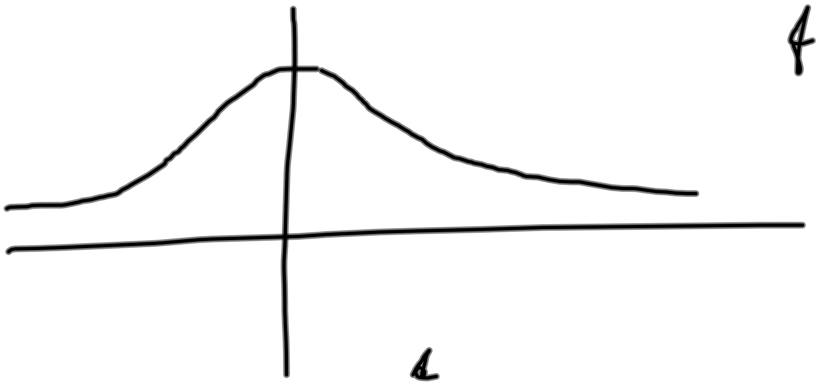
$$f_Y(t) = \begin{cases} 0 & \text{pro } t \le \frac{1}{2} \\ \frac{1}{t^2} & \text{pro } \frac{1}{2} < t < 1 \\ 0 & \text{pro } t > 1 \end{cases}$$

$$P\left[\frac{1}{X} > \frac{2}{3}\right] = P\left[Y > \frac{2}{3}\right] = 1 - P\left[Y \le \frac{2}{3}\right] = 1 - F_Y\left(\frac{2}{3}\right) =$$

$$= 1 - \left(2 - \frac{3}{2}\right) = \frac{1}{2}$$

$$\parallel$$

$$P\left[X < \frac{3}{2}\right] = \frac{1}{2}$$



$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$F_x(t) = \int_{-\infty}^{t} f(y)\,dy \qquad \int_{-\infty}^{\infty} f(y)\,dy = 1$$

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