zásah 1. … $\frac{4}{5}$

— ״ — 2. $\frac{3}{5}$

---

B… v terči se našli 2 zásahy

A… v —— ״ —— oba pocházejí 1. střelba

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B_5)}{P(B)} \doteq 0{,}38$$

$$P(B) = \sum_{i=1}^{6} P(B_i) \qquad A = B_5$$

Rozdělíme si B na sjednocení disjunktních jevů

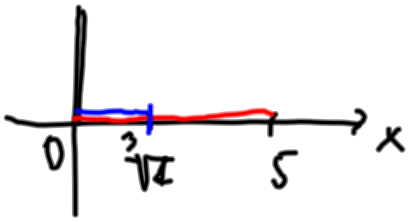
| | 1: 1. | 1: 2. | 2: 1. | 2: 2. | |
|---|---|---|---|---|---|
| $B_1$ | 0 | 1 | 0 | 1 | $\frac{4}{25}\cdot\frac{6}{25}$ |
| $B_2$ | 0 | 1 | 1 | 0 | $\frac{24}{(25)^2}$ |
| $B_3$ | 1 | 0 | 1 | 0 | —״— |
| $B_4$ | 1 | 0 | 0 | 1 | —״— |
| $B_5$ | 1 | 1 | 0 | 0 | $\frac{16}{25}\cdot\frac{4}{25}$ |
| $B_6$ | 0 | 0 | 1 | 1 | [illegible]$/25 \cdot \frac{9}{25}$ |

$Y$... objem krychle

$X$... hrana krychle

$0 \le s \le 125$:

$$F_Y(s) = P[Y \le s] = P[X^3 \le s] = P[X \le \sqrt[3]{s}] = \frac{\sqrt[3]{s}}{5}$$



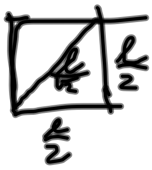

$$F_Y(s) = \begin{cases} 0, & s < 0 \\ \frac{\sqrt[3]{s}}{5}, & 0 \le s \le 125 \\ 1, & s \ge 125 \end{cases}$$

$$f_Y(s) = \begin{cases} 0, & s \le 0 \\ \frac{1}{15} s^{-\frac{2}{3}}, & 0 < s \le 125 \\ 0, & s > 125 \end{cases}$$

Y... délka úhlopříčky obdélníka

X... délka strany obdélníka ($\Rightarrow$ 2. strana $= \ell - X$)

$$\frac{\ell}{\sqrt{2}} \le Y \le \ell$$



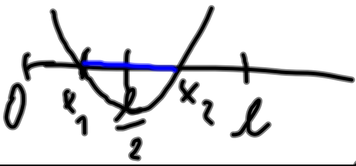

Pro $\frac{\ell}{\sqrt{2}} < s < \ell$:

$$F_Y(s) = P[Y \le s] = P\left[\sqrt{X^2 + (\ell - X)^2} \le s\right] = P\left[X^2 + (\ell-X)^2 \le s^2\right] =$$

$$= P\left[2X^2 - 2\ell X + \ell^2 - s^2 \le 0\right] = \frac{\sqrt{2s^2 - \ell^2}}{\ell}$$

$$2x^2 - 2\ell x + \ell^2 - s^2 = 0 \qquad D = 4\ell^2 - 8(\ell^2 - s^2) = 8s^2 - 4\ell^2$$

$$x_{1,2} = \frac{2\ell \pm 2\sqrt{2s^2 - \ell^2}}{4} = \frac{\ell \pm \sqrt{2s^2 - \ell^2}}{2} \qquad \sqrt{D} = 2\sqrt{2s^2 - \ell^2}$$




$$F_Y(s) = \begin{cases} 0 & s \le \frac{\ell}{\sqrt{2}} \\ \frac{\sqrt{2s^2 - \ell^2}}{\ell}, & \frac{\ell}{\sqrt{2}} \le s < \ell \\ 1, & s \ge \ell \end{cases}$$

Diskrétní náh. veličina nabývá hodnot $x_i$ s pstí $p_i$, $1 \leq i \leq n$

$$EX = \sum_{i=1}^{n} p_i \cdot x_i$$

---

Spojitá náhodná veličina s hustotou $f$:

$$EX = \int_{-\infty}^{\infty} f(x) \cdot x \, dx$$

---

Rozptyl:

$$\operatorname{var} X = E(X - EX)^2 = EX^2 - (EX)^2$$

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$f(x) = \frac{1}{15} x^{-\frac{2}{3}}$ pro $0 \le x \le 125$, 0 jinak

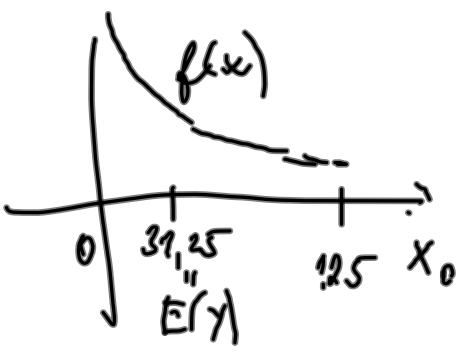
$$EY = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^{125} x \cdot \frac{1}{15} x^{-\frac{2}{3}} = \frac{1}{15} \int_0^{125} x^{\frac{1}{3}} =$$

$$= \left[ \frac{1}{20} x^{\frac{4}{3}} \right]_0^{125} = \frac{5^3}{4} = 31{,}25$$

$\mathrm{var}\, Y \doteq 2232{,}14 - (31{,}25)^2 =$

$\doteq 1255{,}58$

$\sigma Y =$




Medián náhodné veličiny X je hodnota M, taková, že $F_X(M) = \frac{1}{2}$

$$\mathrm{var}\, Y = EX^2 - (EX)^2$$

$$EX^2 = \int_0^{125} x^2 \cdot \frac{1}{15} x^{-\frac{2}{3}}\,dx =$$

$$= \frac{1}{15} \int_0^{125} x^{\frac{4}{3}}\,dx \doteq 2232{,}14$$

$$F_{X^2}(t) = P[X^2 \le t] = P[X \le \sqrt{t}] = F_X(\sqrt{t}) \Rightarrow f_{X^2}(t) = \boxed{\frac{f_X(\sqrt{t})}{2\sqrt{t}}}$$

$$EX^2 = \int_0^2 x \cdot f_{X^2}(t)\, dx =$$

$$= \int_0^2 x \cdot \frac{f_X(\sqrt{x})}{2\sqrt{x}}\, dx = \frac{1}{2}\int_0^2 \sqrt{x}\, f_X(\sqrt{x})\, dx =$$

$$x = t^2$$

$$dx = 2t\, dt$$

$$= \int_0^{\sqrt{2}} t\, f_X(t)\, t\, dt =$$

$$= \int_0^{\sqrt{2}} t^2 f_X(t)\, dt$$

## Normální rozdělení

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \qquad \Phi(t) = \int_{-\infty}^{t} f(t)\,dt$$

$$\left(\int_{-\infty}^{\infty} f(t)\,dt\right)^2 = \int_{-\infty}^{\infty} f(x)\,dx \int_{-\infty}^{\infty} f(y)\,dy =$$

$$\boxed{\iint f(x)\cdot f(y)\,dx\,dy = \int f(x)\,dx \int f(y)\,dy}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}-\frac{y^2}{2}}\,dx\,dy = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} r\,e^{-\frac{r^2}{2}}\,dr\,d\varphi$$

$$\begin{bmatrix} x = r\cos\varphi \\ y = r\sin\varphi \end{bmatrix} \qquad = \int_0^{\infty} r\,e^{-\frac{r^2}{2}}\,dr = \int_0^{\infty} e^{-u}\,du =$$

$$\begin{vmatrix} \cos\varphi & \sin\varphi \\ -r\sin\varphi & r\cos\varphi \end{vmatrix} = r \qquad u = r^2/2, \quad du = r\,dr$$

$$= \lim_{\delta\to\infty}\int_0^{\delta} e^{-u}\,du = \lim_{\delta\to\infty}\left[-e^{-u}\right]_0^{\delta} = 1$$

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$$EX = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x\cdot e^{-\frac{x^2}{2}}\,dx =$$


$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} x\cdot e^{-\frac{x^2}{2}}\,dx + \int_{0}^{\infty} x\cdot e^{-\frac{x^2}{2}}\right] = \frac{1}{\sqrt{2\pi}}\left[\int_{\infty}^{0} e^{-\frac{x^2}{2}}\,dx + \int_{0}^{\infty} x\cdot e^{-\frac{x^2}{2}}\right] =$$

$$\left[\begin{array}{l} s = -x \\ ds = -dx \end{array}\right.$$

$$= 0$$

---

Střední hodnota a rozptyl náh. veličiny s binomickým rozdělením:

$$X \sim Bi(1, p)$$

$$EX = 1\cdot p + 0\cdot(1-p) = p$$

$$\operatorname{var} X = EX^2 - (EX)^2 = p - p^2 = p(1-p)$$

$X \sim Bi(n, p)$

$EX = np$

$\operatorname{var} X = np(1-p)$

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