$EY = y$ -- Y je nestranný odhad pro y

$E\bar{X} = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \bar{x} = \mu = EX_i \ \forall i$

$E S^2 = E\left(\frac{1}{n-1}\sum_{i=1}^{n}(X_i - X)^2\right) = \sigma^2$

$E\bar{X} = E\,\frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{n}\sum_{i=1}^{n} EX_i = \mu$

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$\boxed{\sum_{i=1}^{n}(X_i-\mu)^2} = \sum_{i=1}^{n}\left((X_i - \bar{X}) + (\bar{X}-\mu)\right)^2$

$= \boxed{\sum_{i=1}^{n}(X_i-\bar{X})^2 + n(\bar{X}-\mu)^2} + 2\sum_{i=1}^{n}(X_i-\bar{X})(\bar{X}-\mu)$

$= \text{〃} + \text{〃} + 2(\bar{X}-\mu)\sum_{i=1}^{n}\left(X_i - \frac{1}{n}\sum_{j=1}^{n} X_j\right)$

$= \text{〃} + \text{〃} + \cancel{2(\bar{X}-\mu)\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)}$

$E S^2 = \frac{1}{n-1} E\sum_{j=1}^{n}(X_j-\bar{X})^2 = \frac{1}{n-1} E\sum_{i=1}^{n}(X_i-\mu)^2 - \frac{n}{n-1}E(\bar{X}-\mu)^2$

$= \frac{1}{n-1}\sum_{j=1}^{n}\underbrace{\text{var}\,X_j}_{=\sigma^2} - \frac{n}{n-1}\underbrace{E\,\text{var}\,\bar{X}} = \frac{n}{n-1}\sigma^2 - \frac{n}{n-1}\frac{1}{n}\sigma^2$

$= \sigma^2$

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$\text{var}\,\bar{X} = \text{var}\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{\sigma^2}{n}$

$\text{cov}(W_i, W_j) = E W_i W_j - E W_i\, E W_j$

$E(W_i W_j) = 1\cdot P[W_i = 1, W_j = 1] + 0$

$= \frac{\binom{N-2}{n-2}}{\binom{N}{n}} = \frac{n(n-1)}{N(N-1)}$

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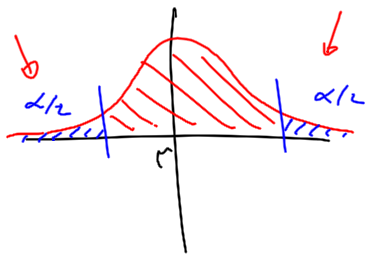

$N(0,1)$

α	0,5	0,10	0,05	0,025
$z(\alpha)$	0	1,282	1,645	1,960

0,01	0,005	0,001
2,326	2,576	3,090

4 30-18:46

Čebyšev:

$\text{var}\,X < \infty$, $\varepsilon > 0$ čt. :

$$P(|X - EX| \geq \varepsilon) \leq \frac{\text{var}\,X}{\varepsilon^2}$$

$\text{var}\,X = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$

$= \int_{|x-\mu|\geq\varepsilon}(x-\mu)^2 f(x)\,dx + \int_{|x-\mu|<\varepsilon}(x-\mu)^2 f(x)\,dx$

$\geq \int_{|x-\mu|\geq\varepsilon}\varepsilon^2 f(x)\,dx = \varepsilon^2 P(|X-EX|\geq\varepsilon)$

4 30-19:14