

$Y$  -- náhodný vektor  
 $Y = X \cdot \beta + \epsilon Z$   
 $Z$  -- náhodná veličina  
 $Z \sim N(0,1)$

$M(X) = \langle x_1, \dots, x_k \rangle$   
 $Y = \hat{Y} + \tilde{Y}$ ,  $\hat{Y} \in M(X)$ ,  $\tilde{Y} \in M(X)^\perp$   
 $\|Y - \hat{Y}\|$  ?

$A \cdot x = b$   
 $X \beta = Y$

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$M(X) = \langle x_1, \dots, x_k \rangle = \langle p_1, \dots, p_k \rangle$   
 $\langle p_1, \dots, p_k, p_{k+1}, \dots, p_n \rangle$  ortonormalní  
 množina v  $\mathbb{R}^n$   
 $P^T \cdot P = E_k$   
 $Q^T \cdot Q = E_{n-k}$

$P = \begin{pmatrix} p_1 & \dots & p_k \end{pmatrix}$   
 $Q = \begin{pmatrix} q_1 & \dots & q_{n-k} \end{pmatrix}$

$V = P^T Z$   $U = Q^T Z$   
 $\hat{Y} = P P^T (X \beta + \epsilon Z)$   
 $= X \beta + \epsilon P P^T Z$   
 $= X \beta + \epsilon P V$   
 $\Rightarrow Y = \underbrace{X \beta + \epsilon P V}_{\hat{Y}} + \underbrace{\epsilon Q U}_{Y - \hat{Y}}$

$P P^T$  je idempotentní a symetrická na  $M(X)$

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$Z = (z_1, \dots, z_n)$  -- náhodná veličina  $Z_i \sim N(0,1)$   
 $Q$  ortonormalní matice  $n \times n$   
 $U = Q^T Z$  -- náhodná veličina  $U_i \sim N(0,1)$

$F(u) = P[U_1 < u_1, \dots, U_n < u_n]$   
 $= \int \dots \int (2\pi)^{-n/2} e^{-\sum z_i^2/2} dz_1 \dots dz_n$   
 $z_i = Q^T z < u_i \Rightarrow z_i = Q^{-1} u_i$   
 $|Q^T| = 1$   
 $= \int \dots \int (2\pi)^{-n/2} e^{-\sum u_i^2/2} du_1 \dots du_n$   
 $= \prod_{i=1}^n \int (2\pi)^{-1/2} e^{-u_i^2/2} du_i$   
 $= \prod_{i=1}^n F(u_i)$  ✓

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$Z = (z_1, \dots, z_n)^T$ ,  $Z_i \sim N(0,1)$  uvažujeme  
 $a \in \mathbb{R}^m$ ,  $B$  matice typu  $m \times n$ ,  $V = B B^T$   
 $U = a + B \cdot Z \sim N_m(a, V)$   
 $m$  -- náhodná veličina vektoru

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$Y = \underbrace{X \beta + \epsilon P V}_{\hat{Y}} + \epsilon Q U$   $V = P^T Z$   $U = Q^T Z$

$\hat{Y} \sim N_k(X \beta, \sigma^2 P P^T)$

$\|Y - \hat{Y}\|^2 / \sigma^2 = \|Q U\|^2 = \|U\|^2 \sim \chi^2_{(n-k)}$

$Y - \hat{Y} \sim N_{n-k}(0, \sigma^2 Q Q^T)$

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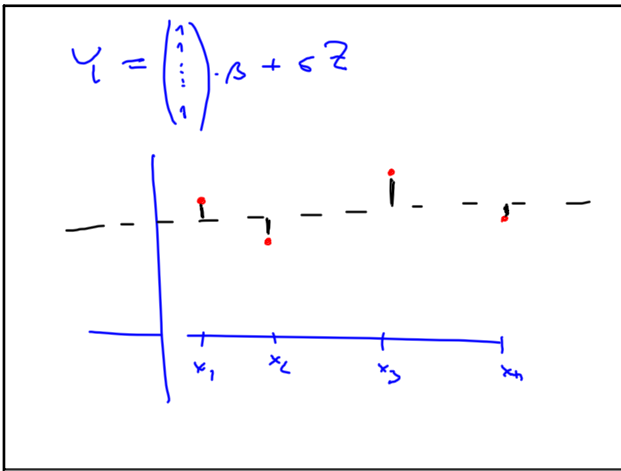
$X \beta = \hat{Y}$   $X^T X \beta = X^T \hat{Y}$   $b = (X^T X)^{-1} X^T \hat{Y}$   
 $X = P T$   
 $b = (T^T P^T P T)^{-1} T^T P^T \hat{Y}$   
 $= T^{-1} (P^T P)^{-1} T^T P^T (P T \beta + \epsilon Z)$   
 $= \beta + \epsilon T^{-1} V$   $V = P^T Z$ ,  $X = P T$

$\Rightarrow \hat{Y} = X b$   
 $\|Y - \hat{Y}\|^2 = \text{RSS}$

$(X^T X)^{-1} = (P^T P T)^{-1} = T^{-1} P^{-1}$  ✓

ostatní vypočítáme  $S^2 = \|Y - X b\|^2 / (n-k)$   
 $b \sim N(\beta, \sigma^2 (X^T X)^{-1})$   $(n-k) S^2 / \sigma^2 \sim \chi^2_{(n-k)}$   
 $E S^2 = \sigma^2$

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