

$(K, \leq)$   $x, y \in K$   $x \wedge y$   $x \vee y$   
 $\Rightarrow$   $f$  ring  
 $\Rightarrow y \in A, \text{ op } A, A \subset K$   
 $\Rightarrow$   $f$  is ring  


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 $x \in f$   
 $f \in \pi$   $f \in \pi \Rightarrow f(f(x)) \in f(\pi)$  **Klein's law**  
 $0 \in f \Rightarrow 0 \in f(x) \in f(f(x)) \in \dots$

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$d(x, y) \leq d(x, z) + d(z, y)$   
 $(x^{n-\ell} \cdot m(x)) : p(x)$   $d(x) \times$   $z$   $f$   $h$   
 $x^{n-\ell} \cdot m(x) = q(x) \cdot p(x) + r(x)$   
 $r(x) + x^{n-\ell} \cdot m(x) = v(x)$   
 $a_0 + a_1 x + \dots + a_n x^n$

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$n=4, \ell=1 \Rightarrow m(x) = b_0$   
 $(x^2 + b_0) : (x^2 + x + 1) = b_0$   $p(x) = 1 + x + x^2$   
 $n-\ell=2$   
 $w=3, \ell=1$   
 $b_0 = x^2 + b_0 x + b_0$   
 $\parallel$   
 $r(x)$   
 $b_0 \mapsto v(x) = b_0 + b_0 x + b_0 x^2$   
 $(1+x)(1+x^2+x^4) = 1 + x + x^2 + x^4 + x^{10} + x^{11}$

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$x^{n-\ell} \cdot m_1(x) = q_1(x) \cdot p(x) + r_1(x)$   
 $x^{n-\ell} \cdot m_2(x) = q_2(x) \cdot p(x) + r_2(x)$   


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 $x^{n-\ell} \cdot (m_1(x) + m_2(x)) = (q_1(x) + q_2(x)) \cdot p(x) + r_1(x) + r_2(x)$

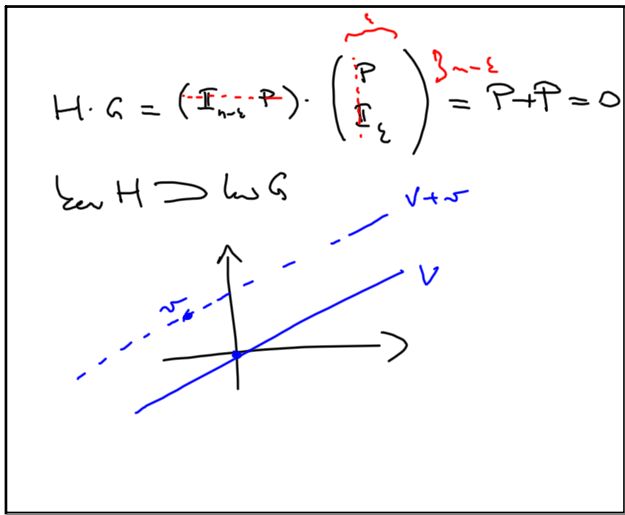
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$x^0 : x^2 : (1+x+x^2) = 1$   $1+x+0+x^2+0x$   
 $x^1 : x^2 + x + 1$   $0+x+x^2+0+x^3$   
 $x^2 : x^4 : (1+x+x^2) = x$   
 $x^4 + x^2 + x$   
 $x^5 : (1+x+x^2) = x^2 + 1$   
 $x^5 + x^3 + x^2$   
 $x^3 + x + 1$   
 $G = \left( \begin{array}{c} P \\ \mathbb{I}_\ell \end{array} \right) \Bigg\}^n$   
 $G : \mathbb{Z}_2^{n-\ell} \rightarrow \mathbb{Z}_2^n$

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$H = \left( \underbrace{\mathbb{I}_{n-\ell} \quad P}_n \right) \Bigg\}^{n-\ell}$   
 $H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$   
 $\ker H = \ker G$   
 $\mathbb{Z}_2^\ell \xrightarrow{G} \mathbb{Z}_2^n \xrightarrow{H} \mathbb{Z}_2^\ell$

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