$$F_Y(y) = P[Y < y] = P[\mu + \sigma Z < y] = P\left[Z < \frac{y-\mu}{\sigma}\right]$$

$x = \mu + \sigma z$

$dx = \sigma\, dz$

$Z \sim N(0,1)$

4 23-18:13

$z = t^{1/2}$

$\dfrac{dz}{\sqrt{x}} = \tfrac{1}{2} t^{-1/2}\, dt$

$$2\int_0^{\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = \cancel{2}\int_0^{x} \frac{1}{\sqrt{2\pi}}\, \cancel{\tfrac{1}{2}}\, t^{-1/2} e^{-t/2}\, dt$$

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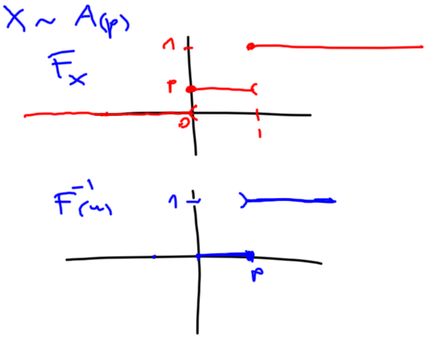
$X \sim A(p)$

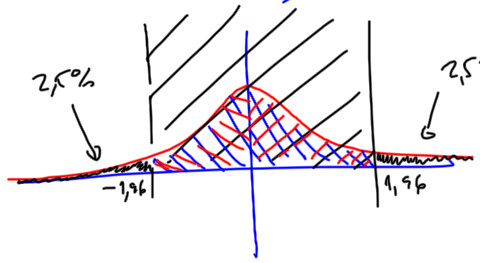


Φ^{-1} j [illegible] [illegible]

4 23-18:23

N.B. $\Phi^{-1}(0{,}975) \doteq 1{,}96$:

$P(|Z| < 1{,}96) = 0{,}95$



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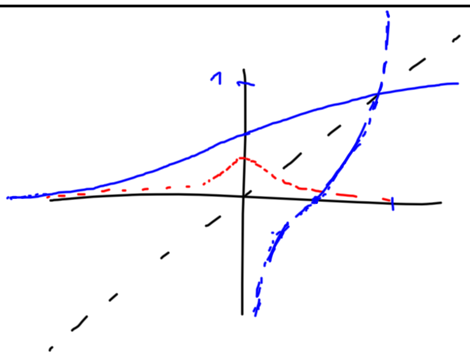


4 23-18:34

$E(XY) = E(X)\,E(Y)$: $P(X=x_i)\cdot P(Y=y_j)$

$$EXY = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_i y_j\, P(X=x_i, Y=y_j)$$

$$= \left(\sum_{i=1}^{\infty} x_i\, P[X=x_i]\right)\left(\sum_{j=1}^{\infty} y_j\, P[Y=y_j]\right)$$

$$= EX \cdot EY$$

4 23-18:41

$$\operatorname{var} X = E(X-EX)^2$$

$$\operatorname{var}(a+bX) = E(a+bX - E(a+bX))^2$$
$$= E(b(X-EX))^2$$
$$= b^2 E(X-EX)^2 = b^2 \operatorname{var} X$$

$$\operatorname{var} X = E(X^2 - 2X\,EX + (EX)^2)$$
$$= EX^2 - 2(EX)^2 + (EX)^2$$

4 23-18:46

$$\operatorname{cov}(X,Y) = E(X-EX)(Y-EY)$$
$$= E(XY - X\,EY - Y\,EX + EX\cdot EY)$$
$$= EXY - EX\cdot EY$$

$$\operatorname{var}(X+Y) = E((X+Y) - E(X+Y))^2$$
$$= E((X-EX) + (Y-EY))^2$$
$$= E((X-EX)^2 + (Y-EY)^2 + 2(X-EX)(Y-EY))$$
$$= \operatorname{var} X + \operatorname{var} Y + 2\operatorname{cov}(X,Y)$$

4 23-18:59

$$|\rho_{X,Y}| \le 1 \qquad \mu_k = E(X-EX)^k$$

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r'$$

kde $\mu_r' = \frac{d^r}{dt^r} M_X(t)\Big|_{t=0}$

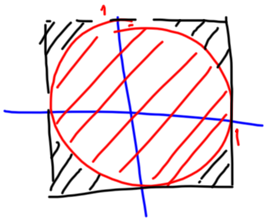
$$W = X+Y$$
$$M_W(t) = E e^{(X+Y)t} = E e^{Xt}\cdot e^{Yt}$$
$$= E e^{Xt}\cdot E e^{Yt} \quad \checkmark$$

4 23-19:06

$$\left(\left(p(e^t-1)+1\right)^n\right)' = n\left(p(e^t-1)+1\right)^{n-1}\cdot p\cdot e^t$$

$$\cdot\Big|_{t=0} = np$$

4 23-19:16



$$X^2+Y^2 \le r^2$$

P závislé

R, Φ plní směrnice

Φ

nezávislé

4 23-19:21

$$\left(1+\frac{x}{n}\right)^n \xrightarrow[n\to\infty]{} e^x$$

X, Y nezávislé, abs. spoj., f_X, f_Y

W = X+Y má za hustotu konvoluci

$$f_W(w) = \int_{-\infty}^{\infty} f_X(x)\, f_Y(w-x)\,dx$$

$$F_W(w) = P[X+Y<w] = \iint_{x+y<w} f_X(x)\, f_Y(y)\,dy\,dx$$
$$= \int_{-\infty}^{w}\left(\int_{-\infty}^{\infty} f_X(x)\, f_Y(t-x)\,dx\right)dt \quad \checkmark$$

4 23-19:30