MA052 / 22

Sparsity motivation

- Vertex separator problem

To make a vertex cut in a graph so that every component has at least 1/3 of vertices. What is the size of the smallest possible cut?

- Problem aliancí v kuize
- Are there symmetries in a graph
    - Long path can disturb symmetries

Lemma:

Every large graph has a nontrivial automorphism, or induced long path, or includes a shallow subdivision of a large complete graph.

shallow topological minor

Lemma

If there is no hommorphism from a triangle to a planar graph G, then there is a hommomorphism from the graph to a triangle.

⇒ Planar graphs are comparable to triangles

MA052/23

Minimum degree: very variable graph property (by removing, it changes)

Average degree: Also variable uppon adding stuff to a graph

Maximum average degree: Stable (over all subgraphs)
→ related to degeneracy
(mad)

<u>k-degenerate graph</u>: we can continuously remove vertices of ~~d~~ degree at most k until the entire graph disappears.

Platí

$$k \geq \lfloor MAD(G) \rfloor$$

$$\Rightarrow G \text{ is } k\text{-degenarate}$$

$$\Rightarrow MAD(G) < 2k$$

$$\frac{MAD(G)}{2} \leq \text{degeneracy} \leq \underline{MAD}(G)$$

<u>Lem</u>: Graph is k-degenerat iff it has an acyclic orientation such that every vertex has in-degree at most k.

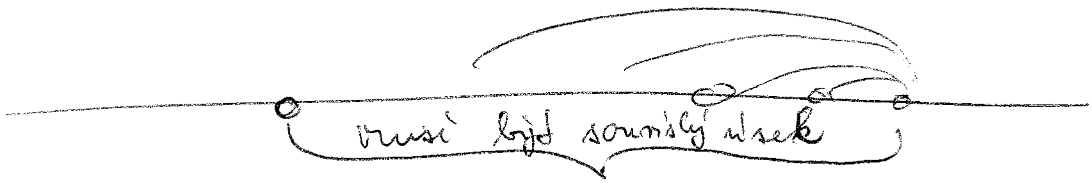
<u>Proof</u>: Hrany odtrhávaného vrcholu orientujeme do něj.

MA052/24

<u>Lemma</u>: Let G be a k-generate graph. Then G includes at most $2^k |V(G)|$ cliques.

<u>Cliques</u>: complete graph

<u>Proof</u>: Odtrháváme vrcholy.



klika k_2 velikosti at most 2^k. Počítáme přes všechny vrcholy.

Girth = Nejkratší cyklus v grafu

Minor: Souvislé podgrafy a mezi nimi hrany

<u>Def</u>: Topological minor F in G:

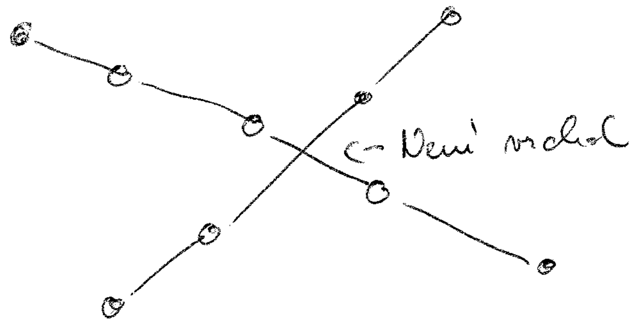
a map $\varphi: V(F) \to V(G)$ injective such that there is a collection of internally disjoint paths $P_{uv} \subseteq G$ for all $uv \in E(F)$.

<u>Def</u>: Immersion F in G:

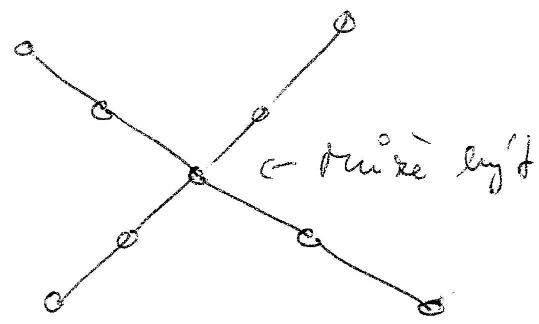
A map $\varphi: V(F)$ in $V(G)$ injective such that there is a collection of <u>edge-disjoint</u> paths $Q_{uv} \subseteq G$, $uv \in E(F)$

MA 052/25

Topologický minor



Immersion



Každý topolický minor je immersion.

<u>Lemma</u> Graph are WQO under immersion.

<u>Lemma</u> Graphs are NOT WQO under topological minors.

(Imerse nezachovávají kreslení na plochy, imerse a minory jsou nesrovnatelné ...)

Ve Sparsity tyto tři pojmy často splývají!