$$3x^2+8xy-3y^2-10=0$$

$$f: U \to \mathbb{R}, \quad u \mapsto f(u)$$

$$\boxed{u\cdot A\cdot u^T}$$

$$(x\ y)\begin{pmatrix} a & b \\ b & d\end{pmatrix}\begin{pmatrix} x \\ y\end{pmatrix}$$

$$(x\ y)\begin{pmatrix} ax+by \\ bx+dy\end{pmatrix} = ax^2+bxy+bxy+dy^2 = ax^2+2bxy+dy^2$$

$$\begin{pmatrix} 3 & 4 \\ 4 & -3\end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda\end{vmatrix} = (3-\lambda)(-3-\lambda)-16 = (\lambda-3)(\lambda+3)-16 = \lambda^2-3^2-16 = \lambda^2-25 = (\lambda-5)(\lambda+5)$$

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$$\begin{pmatrix} 3 & 4 \\ 4 & -3\end{pmatrix} \quad \lambda_1=5 \quad \begin{pmatrix} -2 & 4 \\ 4 & -8\end{pmatrix} \sim \begin{pmatrix} -2 & 4 \\ 0 & 0\end{pmatrix}$$

$$u_1=(2,1)$$

$$\lambda_2=-5 \quad \begin{pmatrix} 8 & 4 \\ 4 & 2\end{pmatrix} \sim \begin{pmatrix} 2 & 1 \\ 0 & 0\end{pmatrix}$$

$$u_2=(1,-2)$$

$$(u_1,u_2)$$

$$\alpha=(v_1,v_2) \quad v_1=\left(\tfrac{2}{\sqrt5}, \tfrac{1}{\sqrt5}\right) \quad v_2=\left(\tfrac{1}{\sqrt5}, \tfrac{-2}{\sqrt5}\right)$$

$$3x^2+8xy-3y^2-10 \qquad (x,y)=(u)_\varepsilon$$

$$\begin{pmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} \\ \frac{1}{\sqrt5} & \frac{-2}{\sqrt5}\end{pmatrix} = (id)_{\varepsilon,\alpha} \qquad \begin{pmatrix} 1 \\ 0\end{pmatrix} \leftarrow (v_1)_\alpha$$

u $S(\varepsilon)_\alpha$

$$(id)_{\varepsilon,\alpha}(u)_\alpha = (u)_\varepsilon$$

$$(a,b)=(u)_\alpha$$

$$\begin{pmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} \\ \frac{1}{\sqrt5} & \frac{-2}{\sqrt5}\end{pmatrix}\begin{pmatrix} a \\ b\end{pmatrix} = \begin{pmatrix} x \\ y\end{pmatrix}$$

$$(id)_{\alpha,\varepsilon} = (id)_{\varepsilon,\alpha}^{-1} = (id)_{\varepsilon,\alpha}^{T}$$

$$x=\frac{2}{\sqrt5}a+\frac{1}{\sqrt5}b = \frac{1}{\sqrt5}(2a+b)$$

$$y=\frac{1}{\sqrt5}a-\frac{2}{\sqrt5}b = \frac{1}{\sqrt5}(a-2b)$$

$$3\,\frac15(2a+b)^2+8\cdot\frac15(2a+b)(a-2b)$$

$$-3\,\frac15(a-2b)^2-10=0 \qquad |\cdot 5$$

$$3\cdot(4a^2+4ab+b^2)+8(2a^2-3ab-2b^2)$$

$$-3(a^2-4ab+4b^2)-50=0$$

$$(12+16-3)a^2+(12-24+12)ab$$

$$(3-16-12)b^2-50=0$$

$$25a^2+0ab-25b^2-50=0$$

$$a^2-b^2-2=0$$

$$(a-b)(a+b)=2$$

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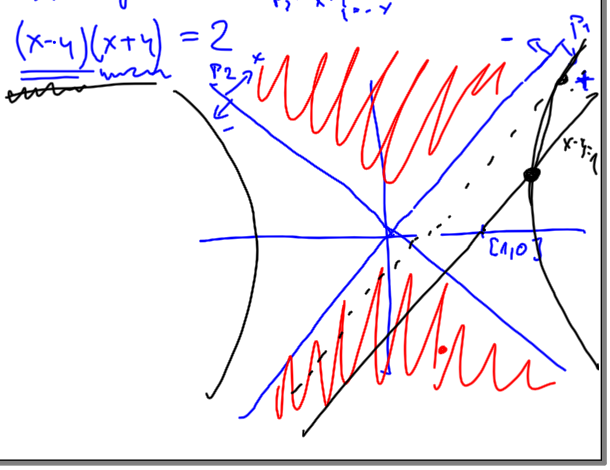
$$x^2-y^2=2 \qquad p_2: x+y=0,\ y=-x \qquad p_1: x-y=0$$

$$(x-y)(x+y)=2$$



5 16-10:36

$$(a-b)(a+b)=2$$

$$\begin{pmatrix} a \\ b\end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt5} & \frac{1}{\sqrt5} \\ \frac{1}{\sqrt5} & \frac{-2}{\sqrt5}\end{pmatrix}\begin{pmatrix} x \\ y\end{pmatrix}$$

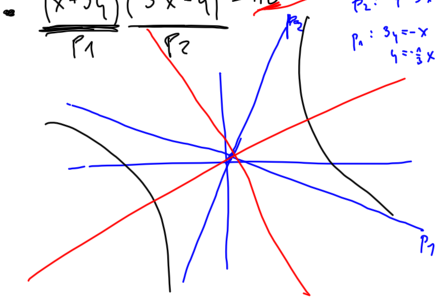
$$a=\frac{1}{\sqrt5}(2x+y) \qquad a-b=x+3y$$

$$b=\frac{1}{\sqrt5}(x-2y)$$

$$\frac{1}{\sqrt5}(x+3y)\,\frac{1}{\sqrt5}(3x-y)=2$$

$$\underbrace{(x+3y)}_{p_1}\underbrace{(3x-y)}_{p_2}=10$$

$$p_2:\ y=3x \qquad p_1:\ 3y=-x,\ y=-\tfrac13 x$$



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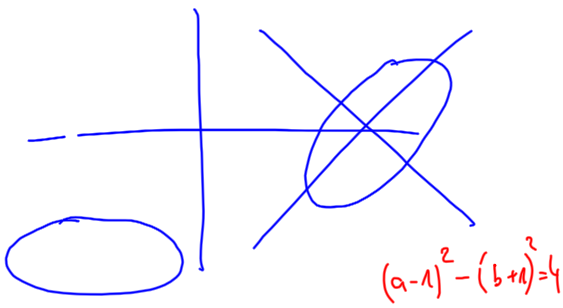

$$(a-1)^2-(b+1)^2=4$$

$$a^2-b^2-2a-2b=4$$

$$(a-1)^2-1-(b+1)^2+1=4$$

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$$A;\quad A\cdot v^T = f(v) \qquad A^T=A$$

$$F(u,v)=u\cdot A\cdot v^T = \langle u, A\cdot v^T\rangle \in \mathbb{R}$$

$$\| \ [u\cdot(A\cdot v^T)]^T \qquad (AB)^T=B^TA^T$$

$$(A\cdot v^T)^T\cdot u^T$$

$$v\cdot A^T\cdot u^T$$

$$v\cdot A\cdot u^T = \langle v, A\cdot u^T\rangle$$

$$\langle u, A\cdot v^T\rangle = \langle v, A\cdot u^T\rangle$$

v ... VL. VEKT PRO VL. Č. λ_1

u ... λ_2

$$\langle u, \lambda_1 v\rangle = \langle v, \lambda_2\cdot u\rangle$$

$$\lambda_1\langle u,v\rangle = \lambda_2\langle v,u\rangle$$

$$(\lambda_1-\lambda_2)\langle u,v\rangle = 0 \qquad \lambda_1\neq\lambda_2$$

$$\Downarrow$$

$$\langle u,v\rangle = 0 \qquad u\perp v$$

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$$3x^2 + 2xy + y^2 + 4yz + 6z^2 =$$

$$3(\cancel{x} + \tfrac{1}{3}y + 0z)^2$$

$$= 3\left(x^2 + \tfrac{2}{3}xy + \tfrac{1}{9}y^2\right) + \underbrace{\tfrac{2}{3}y^2 + 4yz + 6z^2}$$

$$3\left(x + \tfrac{1}{3}y\right)^2 + \underbrace{\tfrac{2}{3}\left(y + 3z\right)^2}$$

$$\tfrac{2}{3}\left(y^2 + 6yz + 9z^2\right) + 0 \cdot z^2$$

$$= 3\left(x + \tfrac{1}{3}y\right)^2 + \tfrac{2}{3}\left(y + 3z\right)^2 + 0 \cdot z^2$$

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$$x^2 + 2xy + y^2 - 2xz - 10yz + z^2$$

$$(x + y - z)^2 - 8yz$$

$$x^2 + y^2 + z^2$$

$$2xy - 2xz - 2yz$$

souř (x, y, z)

$$\begin{cases} x_1 = x + y - z \\ x_2 = y \\ x_3 = z \end{cases}$$

$$x_1^2 - 8x_2x_3$$

$$x_1 = y_1$$
$$x_2 = y_2 + y_3$$
$$x_3 = y_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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