$$p(x) = \begin{cases} \frac{1}{3} & x = -2 \\ \frac{1}{2} & x = 3 \\ \frac{1}{6} & x = 1 \\ 0 & \text{jinak} \end{cases}$$

$E(X)$, $E(2X+5)$, $E(X^2)$, $D(X)$, $D(2X+1)$

$$E(X) = \sum_{-\infty}^{\infty} x\, p(x) = -2\cdot\frac{1}{3} + 3\cdot\frac{1}{2} + 1\cdot\frac{1}{6} = -\frac{2}{3} + \frac{3}{2} + \frac{1}{6} = \frac{-4+9+1}{6} = \frac{6}{6} = 1$$

$$E(aX+b) = aE(X) + b$$

$$E(2X+5) = 2E(X) + 5 = 2\cdot 1 + 5 = 7$$

$$E(X^2) = \sum_{-\infty}^{\infty} x_i^2\, p(x_i) = 4\cdot\frac{1}{3} + 9\cdot\frac{1}{2} + 1\cdot\frac{1}{6} = \frac{4}{3} + \frac{9}{2} + \frac{1}{6} = \frac{8+27+1}{6} = \frac{36}{6} = 6$$

$$D(X) = E\left((X - E(X))^2\right) \; *$$

$$= E(X^2) - E(X)^2 = 6 - 1^2 = 5$$

$$* = E\left((X-1)^2\right) = \sum (x-1)^2 p(x) = (-2-1)^2\cdot\frac{1}{3} + (3-1)^2\cdot\frac{1}{2} + (1-1)^2\frac{1}{6} = 9\cdot\frac{1}{3} + 4\cdot\frac{1}{2} + 0 = 3 + 2 = 5$$

$$D(2X+1) = D(2X) = 2^2\cdot D(X) = 4\cdot D(X) = 4\cdot 5 = 20$$

4 29-17:56

$$C(X,Y) = 0 \qquad Z = 3Y - X$$

$D(X) = a$

$D(Y) = 2$

$a = ?$

$D(Z) = 2$

$$!\; D(X+Y) = D(X) + D(Y) + 2\underbrace{C(X,Y)}_{=0}$$

$$!\; D(X+Y) = D(X) + D(Y)$$

$$D(3Y - X) = D(3Y) + D(-X) = 3^2 D(Y) + (-1)^2 D(X) = 9D(Y) + D(X) = 25$$

$$9\cdot 2 + a = 25$$

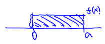
$$18 + a = 25$$

$$a = 7$$

4 29-18:10

X

$$f_X(x) = \begin{cases} \frac{1}{a} & x \in (0,a) \\ 0 & \text{jinak} \end{cases}$$



$$E(2X+3) = 2E(X) + 3$$

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_0^a x\cdot\frac{1}{a}\,dx = \frac{1}{a}\left[\frac{x^2}{2}\right]_0^a = \frac{1}{a}\cdot\frac{a^2}{2} = \frac{a}{2}$$

$$E(2X+3) = 2\cdot\frac{a}{2} + 3 = a + 3$$

$$E(3X^2 - 2X + 1) = E(3X^2) - E(2X) + 1 = 3E(X^2) - 2E(X) + 1$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^a x^2\cdot\frac{1}{a}\,dx = \frac{1}{a}\left[\frac{x^3}{3}\right]_0^a = \frac{a^2}{3}$$

$$E(3X^2 - 2X + 1) = 3\cdot\frac{a^2}{3} - 2\cdot\frac{a}{2} + 1 = a^2 - a + 1$$

$$D(2X+3) = 4\cdot D(X)$$

$$D(X) = E(X^2) - E(X)^2 = \frac{a^2}{3} - \frac{a^2}{4} = \frac{a^2}{12}$$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx}\cdot f_X(x)\,dx = \int_0^a e^{tx}\cdot\frac{1}{a}\,dx = \frac{1}{a}\left[\frac{1}{t}e^{tx}\right]_0^a = \frac{1}{a}\left(\frac{1}{t}e^{ta} - \frac{1}{t}\right) = \frac{1}{at}(e^{ta} - 1)$$

$tx = y$

$t\,dx = dy$

$dx = \frac{dy}{t}$

4 29-18:14

$$M'_X(0) = E(X)$$

$$M''_X(0) = E(X^2)$$

$$\longrightarrow \boxed{M_X(t)\; \frac{1}{at}(e^{ta} - 1)}$$

$$M'_X(t) = \left[\frac{1}{a}\cdot t^{-1}\,(e^{ta} - 1)\right]' = \frac{1}{a}\left[t^{-1}(e^{ta} - 1)\right]' = \frac{1}{a}\left[-1\cdot t^{-2}(e^{ta} - 1) + t^{-1}(e^{ta}\cdot a)\right]$$

$$E(X) \overset{t \to 0}{=}$$

4 29-18:26

$$D(X^2 + 1) = D(X^2) = E((X^2)^2) - E(X^2)^2 = E(X^4) - E(X^2)^2$$

$$E(X^4) = \int_0^a x^4\cdot\frac{1}{a}\,dx = \frac{1}{a}\left[\frac{x^5}{5}\right]_0^a = \frac{a^4}{5}$$

$$D(X^2 + 1) = \frac{a^4}{5} - \frac{a^4}{9} = \frac{4a^4}{45}$$

4 29-18:36

$$\boxed{p(x) = \frac{\lambda^x}{x!}e^{-\lambda}}$$

$$\sum_{-\infty}^{\infty} p(x) = \sum_{-\infty}^{\infty} \frac{\lambda^x}{x!}e^{-\lambda} = e^{-\lambda}\sum_0^{\infty}\frac{\lambda^x}{x!} = e^{-\lambda}\cdot e^{\lambda} = 1$$

$$M_X(t) = E(e^{tX}) = \sum_{k=0}^{\infty} e^{t\cdot k}\cdot$$

$$M_X(t) = E(e^{tX})$$

$$E(Y) = \sum_{-\infty}^{\infty} y\cdot p_Y(y)$$

$$M_X(t) = E(e^{tX}) = \sum_{k=0}^{\infty} e^{tk}\cdot\frac{\lambda^k}{k!}e^{-\lambda} = \sum_{k=0}^{\infty}\frac{\lambda^k\cdot e^{tk}}{k!}e^{-\lambda} = \sum_{k=0}^{\infty} e^{tk}\cdot e^{-\lambda}\cdot\frac{\lambda^k}{k!} = \sum_{k=0}^{\infty} e^{-\lambda}\frac{(e^t\cdot\lambda)^k}{k!} = e^{-\lambda}\sum_{k=0}^{\infty}\frac{(e^t\lambda)^k}{k!} =$$

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!}$$

$$= e^{-\lambda}\cdot e^{e^t\lambda} = e^{e^t\lambda - \lambda} = \boxed{e^{\lambda(e^t - 1)}}$$

4 29-18:39

$M_X(t) = \boxed{e^{\lambda(e^t-1)}}$

$E(X) = M_X'(0)$

$M_X'(t) = e^{\lambda\cdot(e^t-1)} \cdot (\lambda(e^t-1))'$

$= e^{\lambda(e^t-1)} \cdot \lambda \cdot e^t$

$E(X) = M_X'(0) = e^{\lambda\cdot(1-1)} \cdot \lambda \cdot e^0$

$= e^0 \cdot \lambda \cdot e^0 = \underline{\underline{\lambda}}$

$D(X) = \underline{E(X^2)} - E(X)^2$

$E(X^2) = M_X''(0)$

$M_X''(t) = e^{\lambda(e^t-1)} \cdot \lambda e^t \cdot \lambda \cdot e^t + e^{\lambda(e^t-1)} \cdot \lambda \cdot e^t$

$E(X^2) = M_X''(0) = e^{\lambda(1-1)} \cdot \lambda \cdot e^0 \cdot \lambda \cdot e^0 + e^{\lambda\cdot(e^0-1)} \cdot \lambda \cdot e^0$

$= \lambda^2 + \lambda$

$D(X) = E(X^2) - \underline{E(X)^2} = \lambda^2 + \lambda - \lambda^2 = \underline{\underline{\lambda}}$

$E(X) = \sum_{k=0}^{\infty} k \cdot \frac{\lambda^k}{k!} e^{-\lambda} =$

$= e^{-\lambda} \sum_{k=1}^{\infty} k \cdot \frac{\lambda^k}{k!} = e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!} =$

$= \lambda \cdot e^{-\lambda} \cdot \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!}$

$l = k-1$

$= \lambda e^{-\lambda} \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} =$

$= \lambda \cdot e^{-\lambda} \cdot e^{\lambda} = \underline{\underline{\lambda}}$

4 29-18:55

$X \sim Po(120)$ $\quad E(X) = D(X) = \lambda$

$X \in (100, 140)$

$$\boxed{P(|X - E(X)| \geq \varepsilon) \leq \frac{D(X)}{\varepsilon^2}}$$

$P(|X - 120| < 20) \geq 1 - \frac{D(X)}{\varepsilon^2}$

$P(|X - E(X)| \geq 20) \leq \frac{D(X)}{\varepsilon^2}$

$-P(|X - E(X)| \geq 20) \geq -\frac{D(X)}{\varepsilon^2}$

$1 - P(|X - E(X)| \geq 20) \geq 1 - \frac{D(X)}{\varepsilon^2}$

$P(|X - E(X)| < 20) \geq 1 - \frac{D(X)}{\varepsilon^2}$

$E(X) = 120$

$P(|X - 120| < 20) \geq 1 - \frac{120}{400}$

$\underline{P(J) \geq 0{,}7}$

4 29-19:05

$X \sim Bi(4, \frac{2}{3})$

$Y = (X-2)^3$

$p(x) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & x = 0, \dots, n \\ 0 & \text{jinak} \end{cases}$

k	0	1	2	3	4
$p(x)$, $x=k$	$(\frac{1}{3})^4$	$4(\frac{1}{3})^3 \cdot \frac{2}{3}$	$6(\frac{1}{3})^2 \cdot (\frac{2}{3})^2$	$4 \cdot \frac{1}{3}(\frac{2}{3})^3$	$1 \cdot (\frac{2}{3})^4$

$y = (X-2)^2$

$x=k$: $(X-2)^2$	4	1	0	1	4
	$(\frac{1}{3})^4$	$4(\frac{1}{3})^3 \cdot \frac{2}{3}$	$6(\frac{1}{3})^2 \cdot (\frac{2}{3})^2$	$4 \cdot \frac{1}{3}(\frac{2}{3})^3$	$1(\frac{2}{3})^4$

$P_Y(y) = \begin{cases} 6(\frac{1}{3})^2 \cdot (\frac{2}{3})^2 & y = 0 \\ 4 \cdot (\frac{1}{3})^3 \cdot \frac{2}{3} + 4 \cdot \frac{1}{3} \cdot (\frac{2}{3})^3 & y = 1 \\ (\frac{1}{3})^4 + (\frac{2}{3})^4 & y = 4 \end{cases}$

4 29-19:12

X $\quad f(x) = 2x e^{-x^2}$ $\quad \boxed{x > 0}$

$Y = X^2$ $\quad Y = \underline{X^2}$

$X = \sqrt{Y}$ $\quad$ monot. na $(0, \infty)$

$F_X(t) = \int_0^t f(x)\,dx =$

$= \int_0^t \underline{2x \cdot e^{-x^2}}\,dx$

f $\quad \int_a^b f(x)\,dx = F(b) - F(a)$ $\quad$ F je prim f

$F = \int 2x e^{-x^2}\,dx = \left| \begin{array}{l} s = -x^2 \\ ds = -2x\,dx \\ dx = \frac{ds}{-2x} \end{array} \right| =$

$= \int 2x \cdot e^{s} \cdot \frac{ds}{-2x} = -\int e^s\,ds =$

$= -e^s = \underline{-e^{-x^2}}$

$F_X(t) = \int_0^t 2x e^{-x^2}\,dx = \left[-e^{-x^2}\right]_0^t =$

$= -e^{-t^2} + 1 = 1 - e^{-t^2}$

$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x^2} & x > 0 \end{cases}$

$G_Y(y) = P(Y \leq y) = P(X^2 \leq y)$

$= P(X \leq \sqrt{y}) = \underline{1 - e^{-y}}$

$g_Y(y) = e^{-y}$ $\quad y > 0$

4 29-19:18

$X \sim N(68{,}3;\ 0{,}04)$ $\quad \mu = 68{,}3$, $\sigma^2 = 0{,}04$, $\sigma = 0{,}2$

$$U = \frac{X - \mu}{\sigma} \sim N(0,1)$$

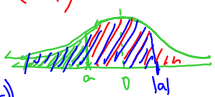
$X \in (68, 69)$

$P(J) = F_X(69) - F_X(68) =$

$F_U\left(\frac{69 - 68{,}3}{0{,}2}\right) - F_U\left(\frac{68 - 68{,}3}{0{,}2}\right)$

$= F_U\left(\frac{0{,}7}{0{,}2}\right) - F_U\left(-\frac{0{,}3}{0{,}2}\right) =$

$= F_U(3{,}5) - F_U(-1{,}5)$



$= F_U(3{,}5) - (1 - F_U(1{,}5)) =$

$= 0{,}99977 - (1 - 0{,}93319) =$

4 29-19:30