$(\mathbb{Z}_8, +) \rightarrow (\mathbb{C}^*, \cdot)$

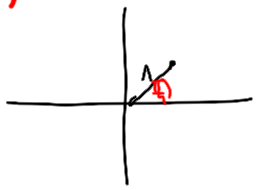
$f: \underline{[a]_8} \rightarrow \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^a$

i) f je zobrazení

$(x,y) \wedge (x,z) \in f \Rightarrow y = z$

$[1]_8 \rightarrow \underline{\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^1}$ $(1+i)$

$[9]_8 \rightarrow \underline{\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^9}$



$\left.\begin{array}{l} \underline{[a]_8} \\ \underline{[a+8k]_8} \end{array}\right\} \Rightarrow$

$z = \underline{\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}}$

$\underline{[a]_8} \rightarrow \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^a$

$\rightarrow \underline{\cos\frac{a\pi}{4} + i\sin\frac{a\pi}{4}}$

$\underline{[a+8k]} \rightarrow \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{a+8k}$

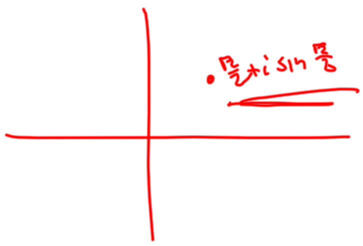
$\rightarrow \cos\frac{(a+8k)\pi}{4} + i\sin\frac{(a+8k)\pi}{4} =$

$= \cos\left(\frac{a\pi}{4} + 2k\pi\right) + i\sin\left(\frac{a\pi}{4} + 2k\pi\right)$

$= \underline{\cos\frac{a\pi}{4} + i\sin\frac{a\pi}{4}}$

$[a+8k]_8 \rightarrow \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^{a+8k}$

$\rightarrow \underline{\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^a} \cdot \underbrace{\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^{8k}}_{\Downarrow\ 1}$



$z: \underline{z^8 = 1}$

$$\text{Inj} \Longleftrightarrow \underline{\underline{\ker = \{e\}}}$$

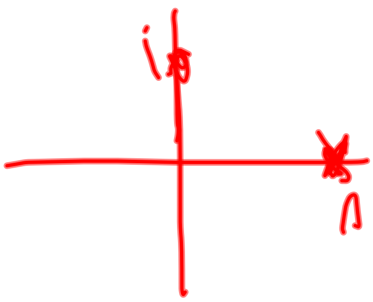
$$\circledcirc \longrightarrow e$$

$$f(a) = 1$$

$$f([a]_6) = 1$$

$$\underbrace{\left(\frac{\sqrt{2}}{2} + i\sin\frac{\sqrt{2}}{2}\right)}^{a} = 1$$

$$\cos\frac{\pi a}{4} + \sin\frac{\pi a}{4} = \cos 0 + i\sin 0$$



$$\frac{\pi a}{4} = 0 + 2k\pi$$

$$\pi a = 8k\pi$$

$$\boxed{a = 8k} \Rightarrow [0]_8$$

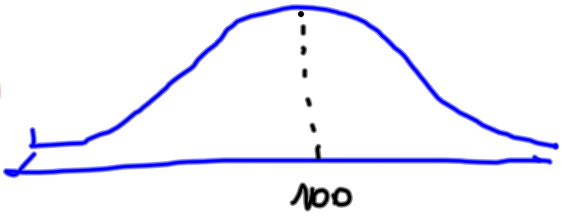
$$\Rightarrow \ker f = \{[0]_8\}$$

$$z = 1 + i$$

$$z = |\sqrt{2}|$$

$IQ \sim N(100, 10^2)$

a) $\underline{P(90 < X < 110)}$



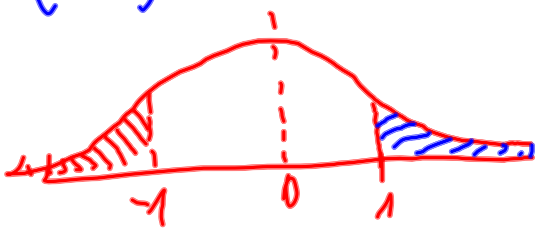
$$\boxed{U = \frac{X - \mu}{\sigma}}$$

$$\underline{U \sim N(0,1)}$$

$$P(90 < X < 110) = F_X(110) - F_X(90)$$

$$= F_U\left(\frac{110-100}{10}\right) - F_U\left(\frac{90-100}{10}\right)$$

$$= F_U(1) - F_U(-1)$$



$1 - F(1)$

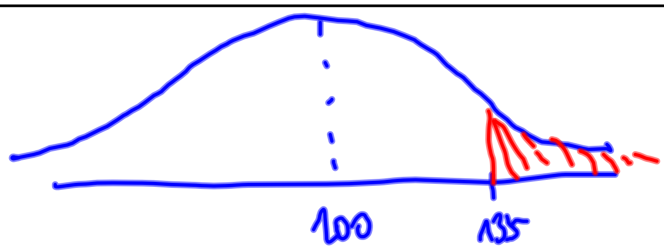
$$= F_U(1) - (1 - F_U(1)) =$$

$$= 2F_U(1) - 1$$

$$= 2 \cdot 0,84135 - 1 = 0,6827$$

$$\doteq 68,27\,\%$$

b) IQ > 135



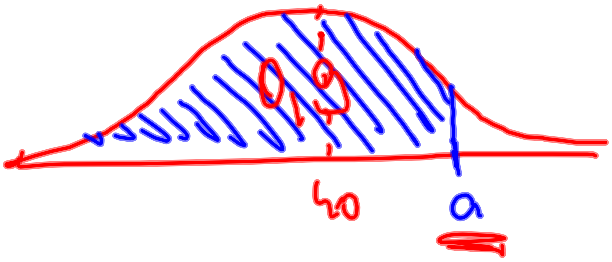
$$P(X \geq 135) = 1 - P(X \leq 135)$$
$$= 1 - F_X(135)$$
$$= 1 - F_U\left(\frac{135-100}{10}\right) =$$
$$= 1 - F_U(3{,}5) =$$
$$= \underline{0{,}00023}$$

$$P(X < 80) = F_X(80) = F_U\left(\frac{80-100}{10}\right) =$$
$$= F_U(-2) = 1 - F_U(2)$$
$$= 0{,}00621$$

$X \sim N(40; 12^2)$ $\quad U = \frac{X-40}{12}$

a) $P(X \leq 45) = F_X(45) = F_U\left(\frac{45-40}{12}\right)$

$= F_U\left(\frac{5}{12}\right) = 0{,}66640$



b) $P(X \leq a) \geq 0{,}9$

$F_X(a) \geq 0{,}9$

$F_U\left(\frac{a-40}{12}\right) \geq 0{,}9$

$\frac{a-40}{12} \geq 1{,}29$

$a - 40 \geq 15{,}48$

$\underline{a \geq 55{,}48}$

$$E(X_1+X_2) = E(X_1)+E(X_2)$$

$$D(X_1+X_2) = D(X_1)+D(X_2)$$

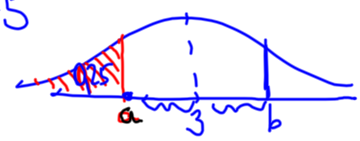
$$Y = 3 + X_1 - 2X_2$$

$$X_1 \sim N(0,1)$$
$$X_2 \sim N(0,1)$$

$$Y \sim N(E(Y), D(X))$$

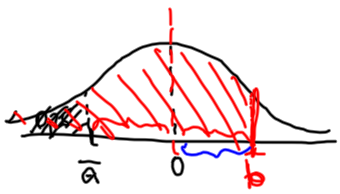
$$E(Y) = E(3+X_1-2X_2) =$$
$$= 3 + \underline{E(X_1)} - 2\cdot \underline{E(X_2)}$$
$$= 3+0-2\cdot 0 = \underline{\underline{3}}$$

$$D(Y) = D(3+X_1-2X_2) =$$
$$= D(X_1) + D(-2X_2)$$
$$= 1 + (-2)^2 D(X_2)$$
$$= 1+4 = 5$$



$$Y \sim N(3,5)$$

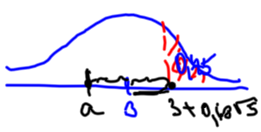
$$F_X(a) = 0{,}25$$



$$F_X(b) = 0{,}75$$

$$F_U\left(\frac{b-3}{\sqrt{5}}\right) = 0{,}75$$

$$\frac{b-3}{\sqrt{5}} \doteq 0{,}68$$



$$b-3 = 0{,}68\cdot\sqrt{5}$$

$$b \doteq 3+0{,}68\cdot\sqrt{5}$$

$$a \doteq 3 - 0{,}68\cdot\sqrt{5}$$

$$\underline{a \doteq 1{,}49}$$

$$3 \quad 4 \quad \underbrace{7\ 7\ 7\ 7}_{4x} \quad \underbrace{8\ 8\ 8\ 8}_{4x} \quad \underbrace{9\ 9\ 9\ 9\ 9\ 9}_{6x}$$

$$\underbrace{10\ 10\ 10}_{3x} \quad 11$$

$$\bar{x} = \frac{1}{n} \sum x_i = 8{,}1$$

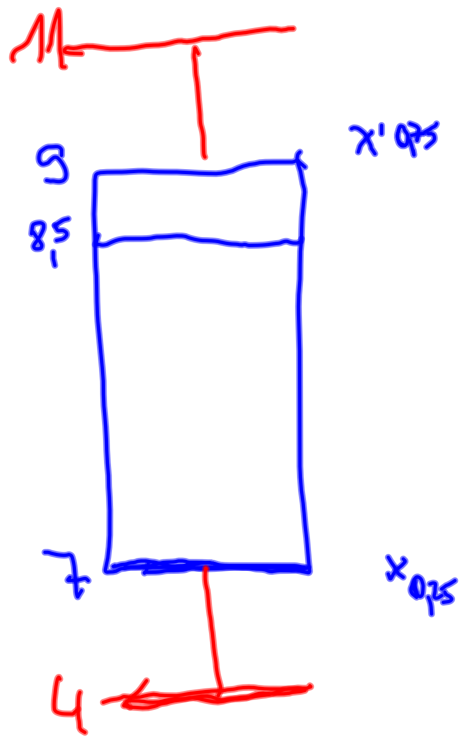
$$\hat{x} = 9$$

$$x_{0,5} = 8{,}5$$

$$x_{0,25} = 7$$

$$x_{0,75} = 9$$

$$Q = 9 - 7 = 2$$



$X \quad E(X) = 3\mu$

$P(X > 3\mu)$

Mark.: $P(X \geq k \cdot E(X)) \leq \frac{1}{k}$

$P(X \geq 3\mu) \leq \frac{1}{3}$

$X \sim Ex\left(\frac{1}{\mu}\right) \qquad f_X = \lambda \cdot e^{-\lambda t}$

$P(X > 3\mu)$

$P(X > 3\mu) = 1 - F_X(3\mu) =$

$1 - \int_0^{3\mu} \frac{1}{\mu} \cdot e^{-\frac{1}{\mu}t}\, dt$

$$= \left| \begin{array}{l} -\frac{1}{\mu}t = u \\ -\frac{1}{\mu}dt = du \\ dt = -\mu\, du \\ \begin{array}{c|c|c} t & 0 & 3\mu \\ \hline u & 0 & -3 \end{array} \end{array} \right| = 1 - \int_0^{-3} \frac{1}{\mu} e^u \cdot (-\mu)\, du$$

$= 1 - \int_{-3}^{0} e^u\, du$

$= 1 - [e^u]_{-3}^{0} = 1 - (1 - e^{-3})$

$= \frac{1}{e^3}$

$n = 600 \qquad P(|X - E(X)| \geq \varepsilon) = \frac{D(X)}{\varepsilon^2}$

$p = \frac{1}{6}$

$P(75 \leq X \leq 125)$

$P(|X - 100| \leq 25) = 1 - \frac{\frac{250}{3}}{625} = 0{,}86$

$E(X) = n \cdot p = 100$

$D(X) = n \cdot p(1-p) = 600 \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{500}{6} = \frac{250}{3}$

5 6-18:58

$n = 64$

$6000\ kg$

$\mu = 90$

$\sigma = 10$

$X \ldots$ hmotnost 64 osob

$E(X) = 64 \cdot 90$

$D(X) = 64 \cdot 100$

$P(X > 6000\ kg) = 1 - F_X(6000)$

$$= 1 - F_U\left(\frac{6000 - 64 \cdot 90}{\sqrt{64 \cdot 10^2}}\right) =$$

$$= 1 - F_U\left(\frac{6000 - 6400 + 640}{80}\right) =$$

$$= 1 - F_U\left(\frac{240}{80}\right) =$$

$$= 1 - F_U(3)$$

$$= 1 - 0{,}9985$$

$$= 0{,}0015$$

X hm. ch osob

pst, že váh. ch osob na hm menší než 5000kg

$$P(X \leq 5000) = F_X(5000)$$

$$P(X > 6000) = 1 - P(X < 6000)$$

$$1 - F_X(6000)$$

5 6-19:22

8 0,08

9 0,962

8 $\mu = 0,5$

5 6-19:25

DLUH Z MINULA

$$M_X(t) = \frac{1}{at}\left(\underline{e^{ta}} - 1\right)$$

$$= \frac{1}{at} \cdot \left(\sum_{k=0}^{\infty} \frac{(at)^k}{k!} - 1\right)$$

$$\mu = \frac{1}{at} \cdot \left(\sum_{k=1}^{\infty} \frac{(at)^k}{k!}\right)$$

$$\mu = \frac{1}{\cancel{at}} \cdot \frac{\cancel{at}}{1!}$$