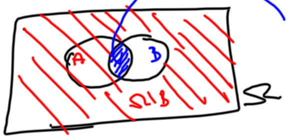


$A\cap B = A\setminus(\Omega\setminus B)$

$x\in A\setminus(\Omega\setminus B) \iff x\in A \wedge x\notin \Omega\setminus B \iff$

$\iff x\in A \wedge x\in B \iff x\in A\cap B$

4 13-14:22

$P(\emptyset)=0$, neboť $\Omega,\emptyset$ jsou neslučitelné $\Rightarrow$

$\Rightarrow P(\underbrace{\Omega\cup\emptyset}_{\Omega}) = P(\Omega)+P(\emptyset)$

$1 = 1 + P(\emptyset) \Rightarrow P(\emptyset)=0$

- $0\le P(A)\le 1$: $\underbrace{P(A\cup(\Omega\setminus A))}_{1=P(\Omega)} = P(A)+\underbrace{P(\Omega\setminus A)}_{\ge 0}$

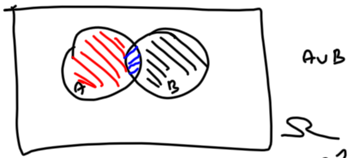
$\Rightarrow P(A)\le 1$, navíc $P(A^c)=1-P(A)$

- $A\subseteq B$: $B = A\cup(B\setminus A)$, $A, B\setminus A$ neslučitelné

$P(B) = P(A)+\underbrace{P(B\setminus A)}_{\ge 0} \Rightarrow P(A)\le P(B)$; $P(B\setminus A) = P(B)-P(A)$

4 13-14:30

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$



$A\cup B = (A\setminus B)\cup(B\setminus A)\cup(A\cap B)$ po 2 neslučitelné

$= A\cup(B\setminus A) = A\cup(B\setminus(A\cap B))$, $A\cap B\subseteq B$

$\Rightarrow P(A\cup B) = P(A)+P(B\setminus A) =$

$= P(A)+(P(B)-P(A\cap B))$

4 13-14:36

Tvrzení: $A_1\subseteq A_2\subseteq A_3\subseteq\dots\subseteq A_n\subseteq\dots$

$\bigcup_{i=1}^{\infty} A_i = A_1\cup(A_2\setminus A_1)\cup(A_3\setminus A_2)\cup\dots\cup(A_n\setminus A_{n-1})\cup\dots$

po 2 neslučitelné ✓

z def.:

$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=2}^{\infty} P(A_i\setminus A_{i-1}) + P(A_1) =$

$= \sum_{i=2}^{\infty}\left(P(A_i)-P(A_{i-1})\right) + P(A_1)$

$= \lim_{n\to\infty}\left(\sum_{i=2}^{n} P(A_i)-P(A_{i-1})\right)+P(A_1) =$

$= \lim_{n\to\infty}\left(P(A_n)-P(A_1)\right) + P(A_1) =$

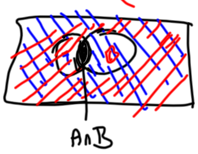
$= \lim P(A_n)$

4 13-14:40

- $\bigcap_{i=1}^{\infty} A_i = \left(\bigcup_{i=1}^{\infty} A_i^c\right)^c$

de Morgan

$A\cap B = (A^c\cup B^c)^c$



$A\cap B$

$P\left(\bigcap_{i=1}^{\infty} A_i\right) = P\left(\left(\bigcup_{i=1}^{\infty} A_i^c\right)^c\right) = 1 - P\left(\bigcup A_i^c\right)$

$= 1-\lim_{n\to\infty} P(A_n^c) = 1-\lim_{n\to\infty}(1-P(A_n)) =$

$= 1-1+\lim_{n\to\infty}P(A_n)$

4 13-14:47

$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P(A_1)+P(A_2\setminus A_1)+\dots+P(A_n\setminus A_{n-1})+\dots$

$\le P(A_1)+P(A_2)+\dots+P(A_n)+\dots$

$P\left(\bigcap_{i=1}^{\infty} A_i\right) = 1-P\left(\bigcup_{i=1}^{\infty} A_i^c\right) \ge 1-\left(P(A_1^c)+\dots\right)$

$= 1-\sum_{i=1}^{\infty}\left(1-P(A_i)\right)$

4 13-14:53

$$P(A|B) = \frac{P(A)\cdot P(B|A)}{P(B)}$$

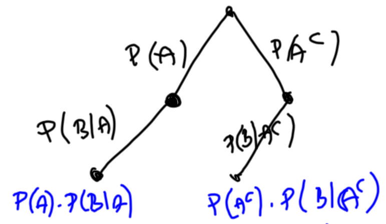
$$P(B) = P(A)P(B|A) + P(A^c)\cdot P(B|A^c)$$

$H_1,\dots,H_k$ neslučitelné $H_1 \cup \dots \cup H_k = \Omega$

$$P(B) = \sum_{i=1}^{k} P(H_i)\cdot P(B|H_i)$$

$$P(H_c|A) = \frac{P(A|H_c)\cdot P(H_c)}{\sum_{i=1}^{k} P(H_i)\cdot P(A|H_i)}$$

4 13-15:14



$$P(A|B) = \frac{P(A)\cdot P(B|A)}{P(A)\cdot P(B|A) + P(A^c)\,P(B|A^c)}$$

4 13-15:17