$\{x_i\}, \{y_i\}$ posloupnosti v X, Cauchyovské

$$\sim \; (\equiv) \; d(x_i, y_i) \xrightarrow[i \to \infty]{} 0$$

je to ekvivalence

$$\{x_i\} \sim \{y_i\}, \; \{y_i\} \sim \{z_i\}$$

$$d(x_i, z_i) \leq \underbrace{d(x_i, y_i)}_{\substack{\downarrow \, i \to \infty \\ 0}} + \underbrace{d(y_i, z_i)}_{\substack{\downarrow \, i \to \infty \\ 0}} \longrightarrow 0$$

$$X \ni x = \{x_i = x\} \hookrightarrow \tilde{X}$$

$$\boxed{\tilde{d}([x_i], [y_i]) = \lim_{i \to \infty} d(x_i, y_i)}$$

$\forall \varepsilon \; \exists N, \; i, j > N \quad d(x_i, x_j) < \varepsilon, \; d(y_i, y_j) < \varepsilon$

$$|d(x_i, y_i) - d(x_j, y_j)| \leq$$

$$\leq |d(x_i, y_i) - d(x_j, y_i)| + |d(x_j, y_i) - d(x_j, y_j)|$$

$$\leq |d(x_i, x_j)| + |d(y_i, y_j)| \leq 2\varepsilon$$

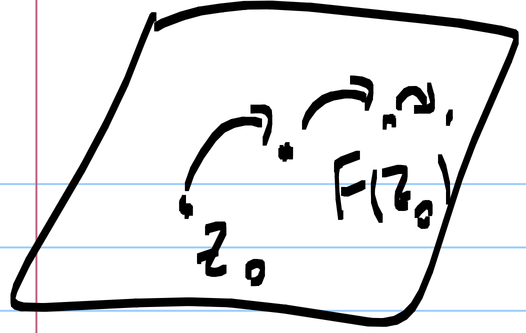
$$d(x,y) + d(y,z) \geq d(x,z)$$

$$d(x,y) \geq d(x,z) - d(y,z)$$

1) $\tilde{d}$ je dobře definovaná metrika na $\tilde{X}$

2) $\boxed{\tilde{X} \text{ úplný}}$

3) $X \subset \tilde{X}$ je hustá podmnožina



X

$$z_0,\ z_1 = F(z_0),\ z_2 = F(z_1),\ \dots$$

$$d(z_i, z_{i+1}) = d(F(z_{i-1}), F(z_i))$$

$$\le C\, d(z_i, z_{i-1}) \le \dots \le C^i d(z_1, z_0)$$

$0 \le C < 1$

$$d(z_{i+j}, z_i) \le \sum_{k=1}^{j} d(z_{i+k}, z_{i+k-1})$$

$$\le \sum_{k=1}^{j} C^{i+k-1} d(z_1, z_0) = C^i d(z_1, z_0) \sum_{k=1}^{j} C^{k-1}$$

$$\le \frac{C^i}{1-C} d(z_1, z_0) < \varepsilon \text{ for } i > N$$

$\{z_i\}$ ist Cauchyfolge $\Rightarrow$ ex. $z_\infty = \lim_{i\to\infty} z_i$

$$F(z_\infty) = \lim_{i\to\infty} F(z_i) = z_\infty .$$

$z_i \in A_i$ $\qquad A_1 \supset A_2 \supset A_3 \ldots$

$\operatorname{diam} A_i \to 0$

$\forall \varepsilon > 0$, $\exists n(\varepsilon)$, h. ha $i > n(\varepsilon)$ $\operatorname{diam} A_i < \varepsilon$

$\Rightarrow j > i > n(\varepsilon)$ $\quad d(z_i, z_j) \le \varepsilon$

$\Rightarrow \{z_i\}$ Cauchy-sorozat $\Rightarrow$ $\exists$ limesz $\underline{z}$

z is limesze lesz mind A_i, így $z \in A_i$

$\Rightarrow z \in \bigcap A_i$.

Végül $w \in \bigcap A_i$ $\quad d(z, w)$ is [illegible] 0. $\boxed{\text{Z}}$

$A_i \subset X$ otevřené, husté. $U \subset X$ otevř.

$$\left(\bigcap_{i=1}^{\infty} A_i\right) \cap U \neq \emptyset \quad ?$$

$z_1 \in A_1 \cap U \qquad z_1 \in \overline{O_{\varepsilon_1}(z_1)} \subset A_1 \cap U$

induktivně: $z_i \in O_{\varepsilon_i}(z_i) = U_i, \; i = 1, \dots, n \Rightarrow \exists\, z_{n+1} \in A_{n+1} \cap \overline{U_n}$

$B_{n+1} = \overline{U_{n+1}} \subset \overline{U_n}, \quad B_{n+1} \subset A_{n+1} \cap \overline{U_n}, \quad \varepsilon_n \leq 1/n$

$$z \in \bigcap_{i=1}^{\infty} \overline{U_i} = \bigcap_{i=1}^{\infty} B_i \subset \left(\bigcap_{i=1}^{\infty} A_i\right) \cap U$$

<u>Věta</u> (Arzelaova-Ascoliho)

$M \subset C[a,b]$ je kompaktní $\Longleftrightarrow$
- omezené
- uzavřené
- stejnoměrně spojité