$$S^{0}[a,b] \qquad \|f\|_p = \left(\int_a^b |f(x)|^p\,dx\right)^{1/p}$$

$$p=2: \quad \langle f, g\rangle = \int_a^b f(x)\,\overline{g(x)}\,dx$$

$$\Rightarrow \quad \langle f, f\rangle = \|f\|_2^2 = \int_a^b |f|^2\,dx$$

$a = -\pi$, $b = +\pi$

periodické i periodo 2π

$$f \longmapsto F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

obecné a, b $\qquad \omega = \frac{2\pi}{T}$

$$\boxed{a_n = \frac{2}{T}\int_{x_0}^{x_0+T} f(x)\cos n\omega x\,dx} \qquad \boxed{b_n = \frac{2}{T}\int_{x_0}^{x_0+T} f(x)\sin n\omega x\,dx}$$

$T = b - a$

$f(x),\ s_N(x)$

$$\int_a^b |f(x)| \cdot 1 \, dx \le |b-a|^{1/q} \left(\int_a^b |f(x)|^p \, dx \right)^{1/p} \le |b-a|^{1/q} C^{1/q} \left(\int_a^b |f(x)| \, dx \right)^{1/p}$$

C^{p-1}

L_1

$$|f(x)| \le C \qquad |f(x)|^p \le C^{p-1} |f(x)| \qquad \frac{1}{p}(p-1) = 1/q$$

$$a_n = \frac{2}{T}\int_a^b f(x)\cos \omega n x \, dx \quad , \; b_n = \ldots \qquad \omega = \frac{2\pi}{T}$$

$$\boxed{|a_n| \leq \frac{2}{T}\int_a^b |f(x)| \, dx}$$

per partis . $T = 2\pi$

$$a_n(f) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx = \frac{1}{n\pi}\Big[\overset{=0}{f(x)\sin nx}\Big]_{-\pi}^{\pi} - \frac{1}{n\pi}\int_{-\pi}^{\pi} f'(x)\sin nx \, dx$$

$$= \frac{1}{n} b_n(f')$$

$$\Rightarrow |a_n(f)| \leq \frac{1}{n^{k+1}\pi}\int_{-\pi}^{\pi} |f^{(k+1)}(x)| \, dx$$

$f(x) \in S^1[a,b]$

$$|S_N(x) - f(x)| = \left|\sum_{k=N+1}^{\infty} \left(a_k \cos(kx) + b_k \sin(kx)\right)\right| \le \sum_{k=N+1}^{\infty} (|a_k| + |b_k|)$$

$$|S_N(x) - f(x)| \le \sum_{k=N+1}^{\infty} \frac{1}{k}(|a_k'| + |b_k'|) \le \left(2\sum_{k=N+1}^{\infty} \frac{1}{k^2}\right)^{1/2} \cdot \left(\sum_{k=N+1}^{\infty} (|a_k'|^2 + |b_k'|^2)\right)^{1/2}$$

H.n. $p = q = 2$

$$\le \sqrt{2}\left(\int_N^{\infty} \frac{1}{x^2}\,dx\right)^{1/2} \cdot \frac{1}{\sqrt{\pi}} \|f'\|_2$$

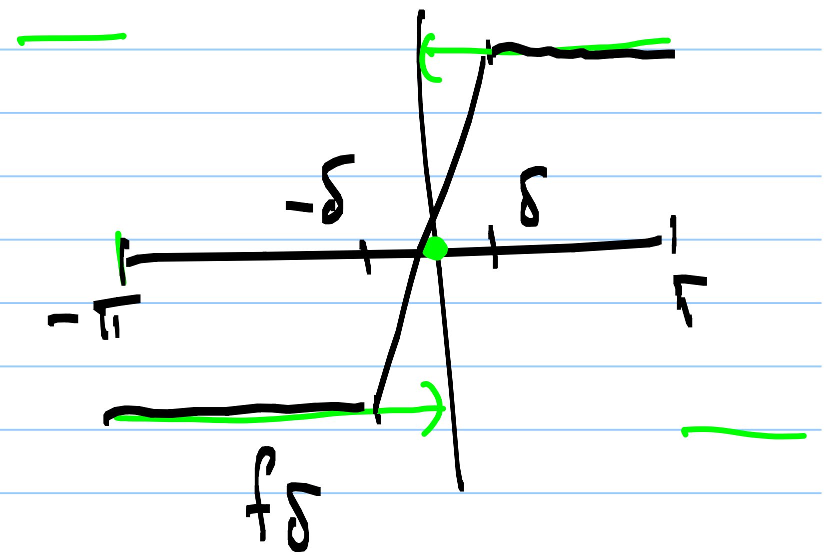
$$= \left(\frac{\sqrt{2}}{\sqrt{\pi}} \|f'\|_2\right) \cdot \frac{1}{\sqrt{N}}$$

L_2 - [illegible]:

Lemma: $C^0[a,b] \subset S^0[a,b]$ je hustá v L_2.

proč?

$$h = \begin{cases} -1 & \text{pro } x < 0 \\ 0 & \text{pro } x = 0 \\ 1 & \text{pro } x > 0 \end{cases}$$



$$\| h - f_\delta \|_2 \to 0$$

$$\|f - S_N(f)\|_2 \le \underbrace{\|f - f_\varepsilon\|_2}_{\le \varepsilon} + \underbrace{\|f_\varepsilon - S_N(f_\varepsilon)\|_2}_{\le \varepsilon \text{ pro velké } N} + \underbrace{\|S_N(f_\varepsilon) - S_N(f)\|_2}_{?}$$

$$\|S_N(f - f_\varepsilon)\|_2 \le 2\,\|f - f_\varepsilon\|_2 \le 2\varepsilon$$

$$\Rightarrow \|f - S_N(f)\|_2 \le 4\varepsilon .$$

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, e^{-i\omega_n t}\, dt$$

$$c_n = \frac{1}{2}(a_n - i b_n) \qquad c_{-n} = \frac{1}{2}(a_n + i b_n)$$

$$\boxed{F(t) = \sum_{n=-\infty}^{\infty} c_n e^{i\omega_n t}}$$

$$\int_{-T/2}^{T/2} K_N(x)\,dx = 1.$$

$$s_N(x) = \int_{-T/2}^{T/2} K_N(y)\, f(x+y)\,dy$$

$$s_N(x) - f(x) = \int_{-T/2}^{T/2} \left(f(x+y) - f(x)\right) K_N(y)\,dy$$

$$\frac{f(x+y) - f(x)}{\sin(\omega y/2)} \quad \sin\left(\left(N+1/2\right)\omega y\right) = \varphi_x(y)\left[\cos(\omega y/2)\sin(N\omega y) + \sin(\omega y/2)\cos(N\omega y)\right.$$

$S_N(x) - f(x)$ is sum of Fourier coeff. of

for ψ_1, ψ_2 $\quad (b_N(\psi_1), a_N(\psi_2))$

$$\psi_1(y) = \frac{1}{2}\varphi_x(y)\cos(Ny/2),$$

$$\psi_2(y) = \frac{1}{2}\varphi_x(y)\sin(Ny/2)$$

so $N \to \infty$ goes $\to 0$

we've shown for $f \in C^1[a,b]$

$x=0$ we [illegible]

$$f(x) = \underbrace{\tfrac{1}{2}\left(f(x) + f(-x)\right)}_{f_1} + \underbrace{\tfrac{1}{2}\left(f(x) - f(-x)\right)}_{f_2}$$

$$f_1(0) = \frac{1}{2}\left(\lim_{y \to 0_+}\left(f(y)\right) + \lim_{y \to 0_-}\left(f(y)\right)\right) \quad \ldots$$