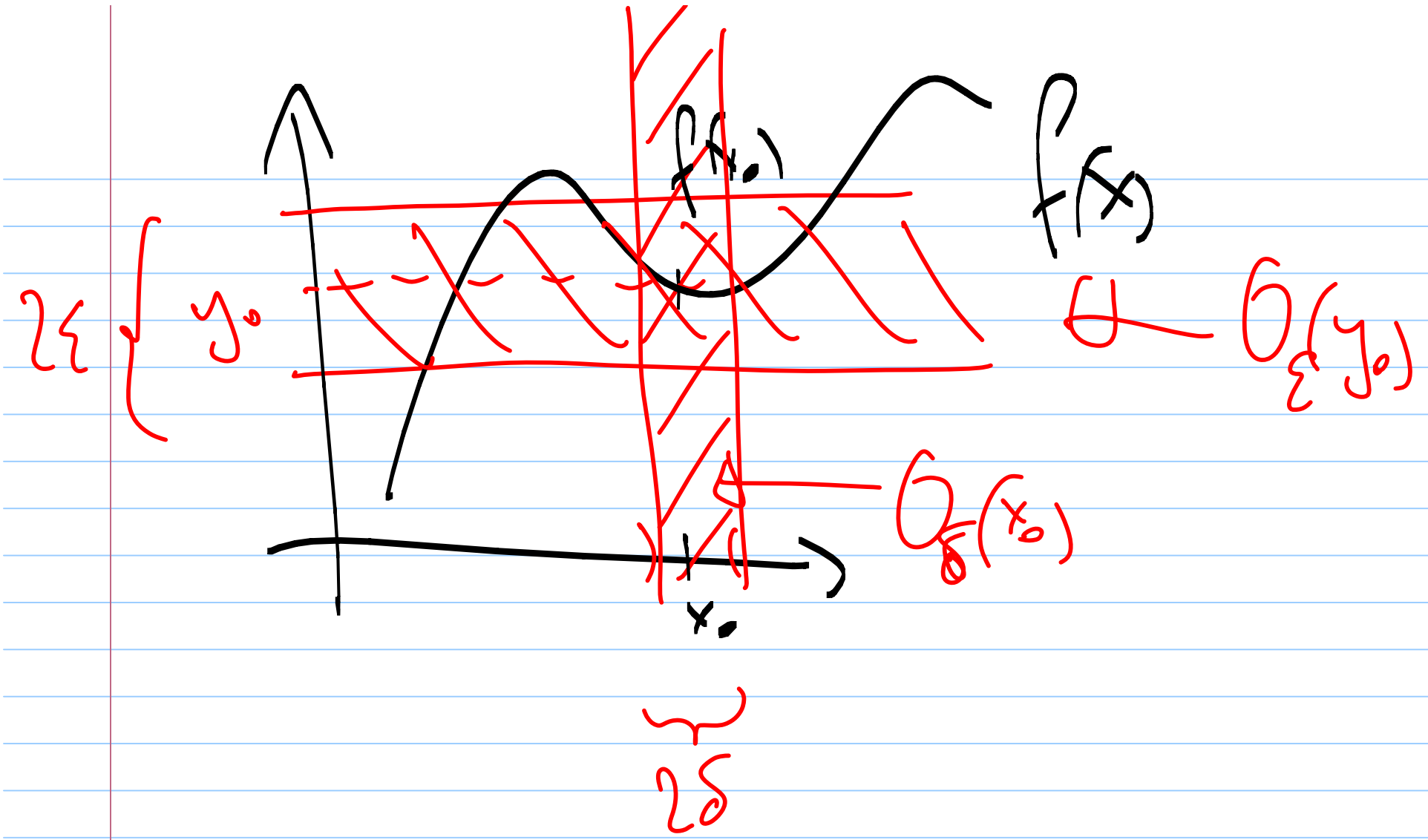
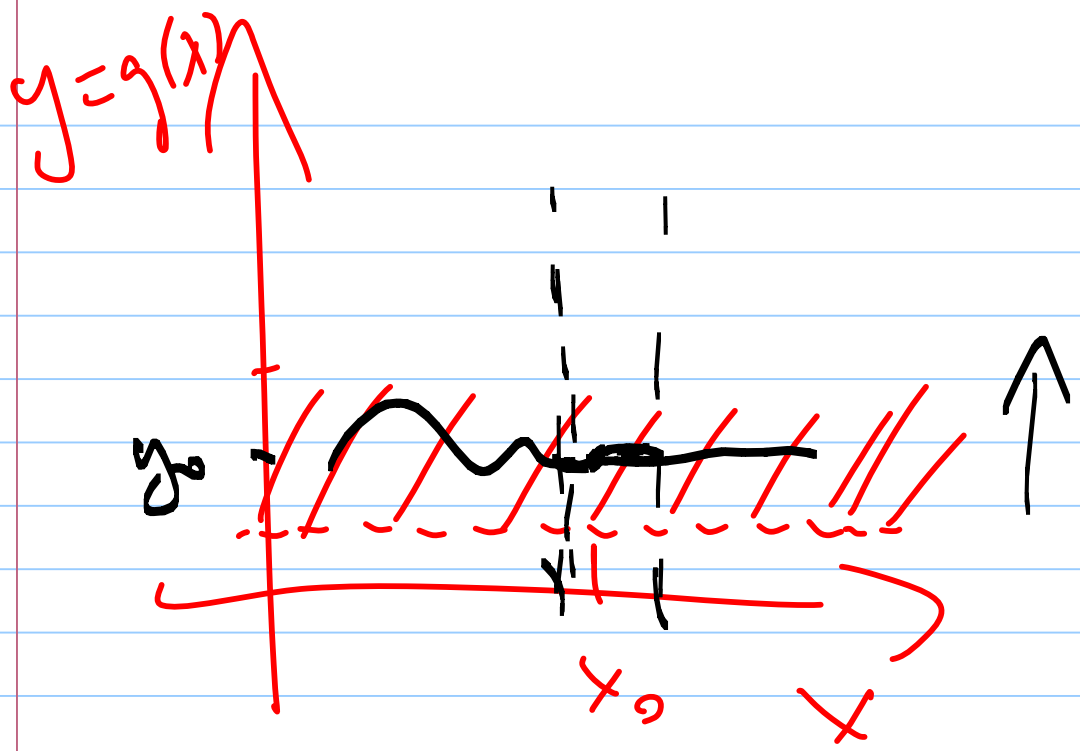
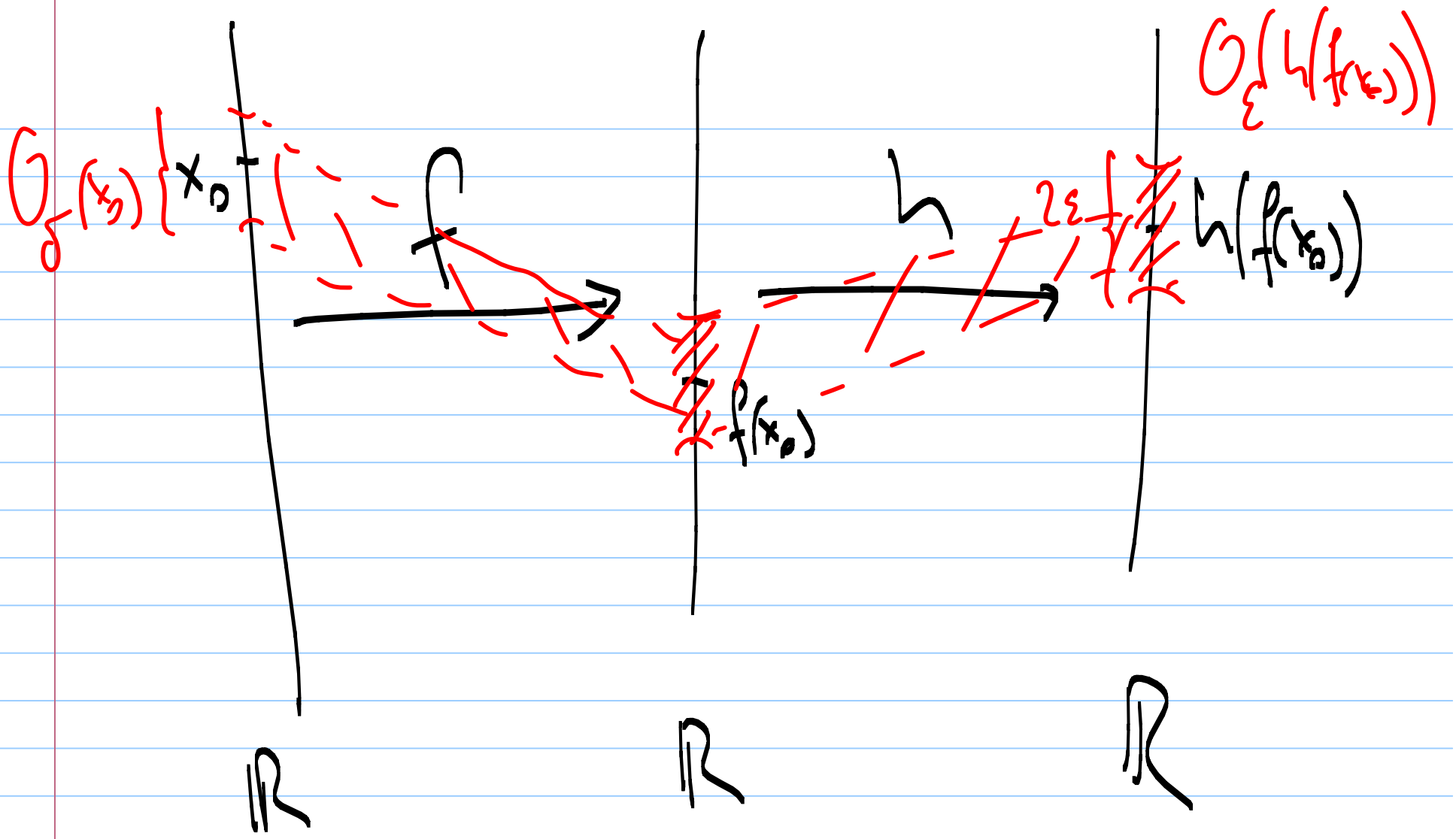
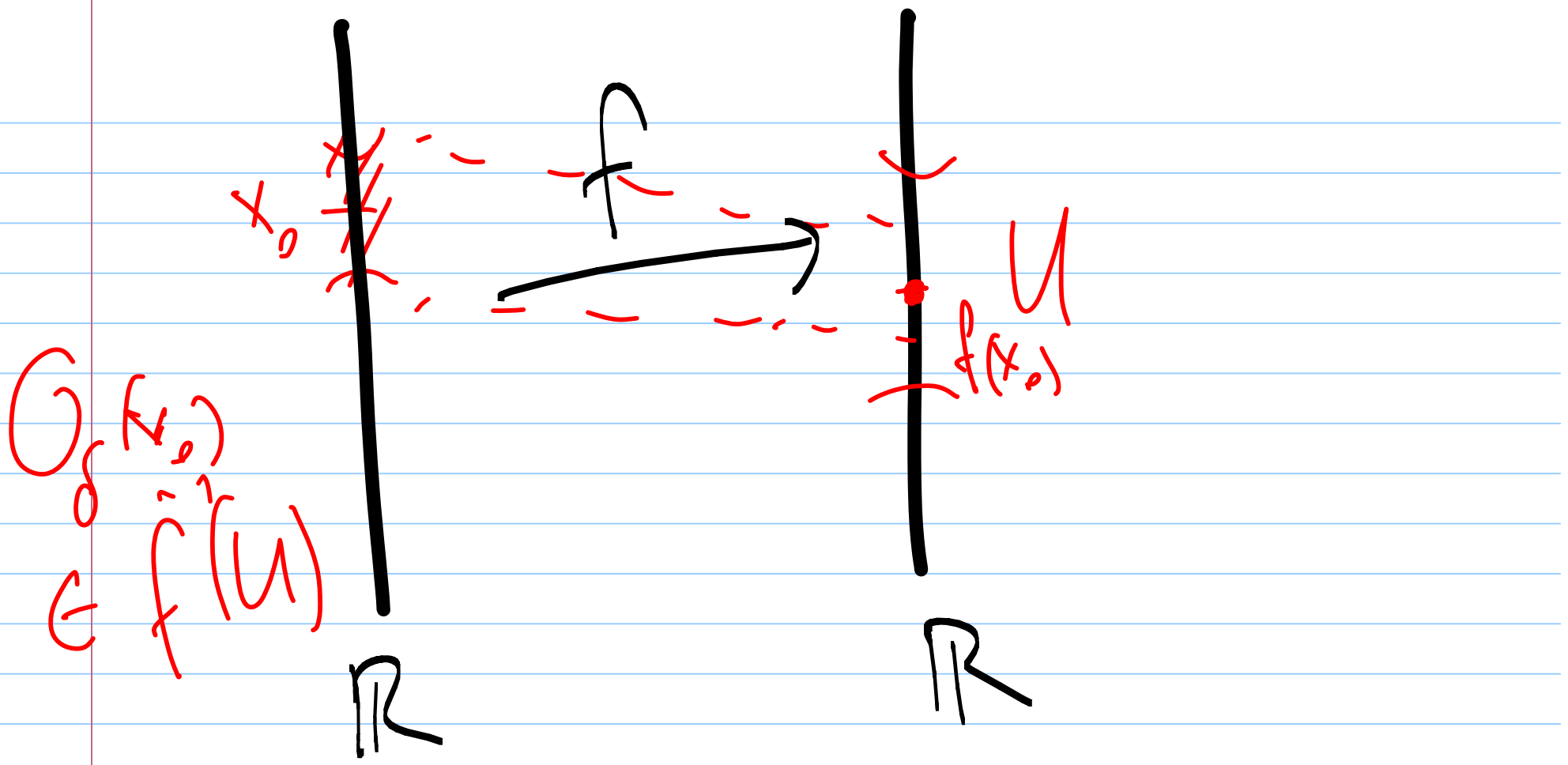
Spojitá limita $x \to x_0 \pm$

$$f(x_0) = \lim_{x \to x_0} f(x)$$

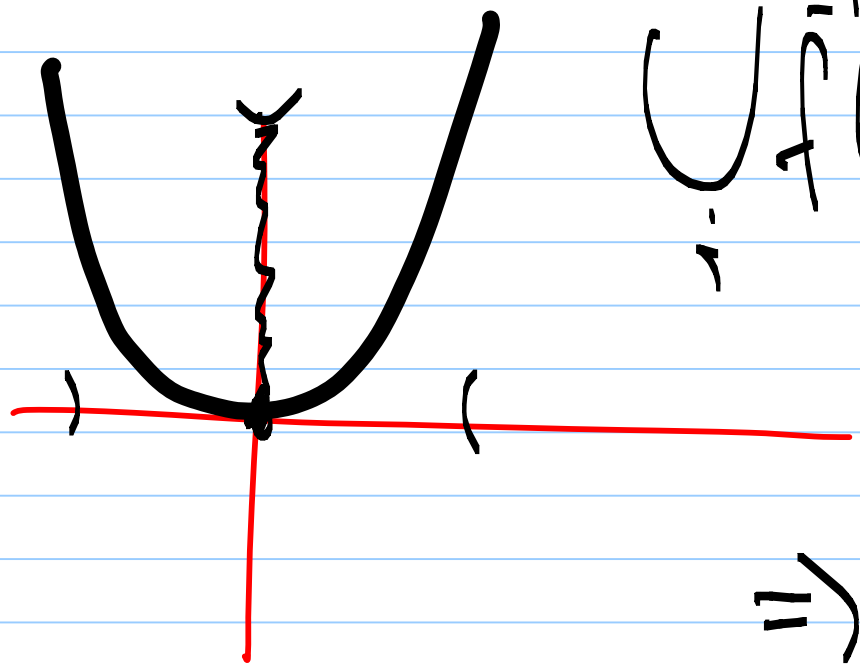

f(x₀)
f(x)
2ε
y₀
U
G_ε(y₀)
G_δ(x₀)
x₀
2δ


y=g(x)
y₀
x₀
x


$O_\delta(x_0)$
x_0
f
$f(x_0)$
h
2ε
$h(f(x_0))$
$O_\varepsilon(h(f(x_0)))$
$\mathbb{R}$
$\mathbb{R}$
$\mathbb{R}$


x_0
f
U
$f(x_0)$
$O_\delta(x_0)$
$\subset f^{-1}(U)$
$\mathbb{R}$
$\mathbb{R}$

$f(x) = x^2$



$f(K) \subset \bigcup_i U_i$

$\bigcup_i f^{-1}(U_i) \supset K$

$\Rightarrow \bigcup_{j=1}^{k} f^{-1}(U_j) \supset K$

$\Rightarrow \bigcup_{j=1}^{k} U_j \supset f(K)$

$$\lim_{x \to 0} \frac{\sin^2 x}{x} =$$

$$= \lim_{x \to 0} \left(\sin x \cdot \frac{\sin x}{x} \right) = 0 =$$

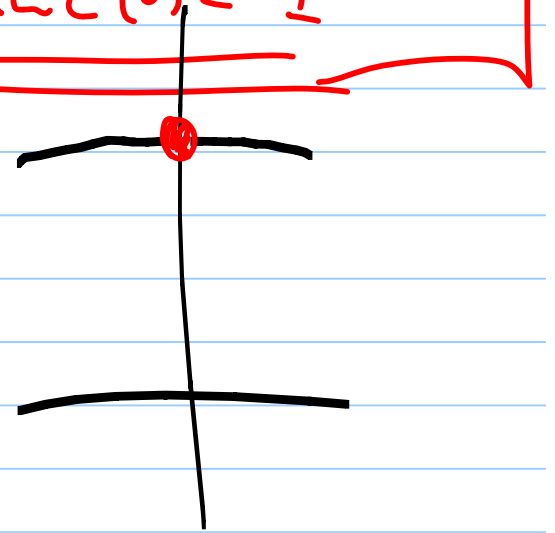
$\sin x \to 0$, $\frac{\sin x}{x} \to 1$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\Vert$$

$\sin c(x) = \frac{\sin x}{x}$

$\sin c(0) = 1$



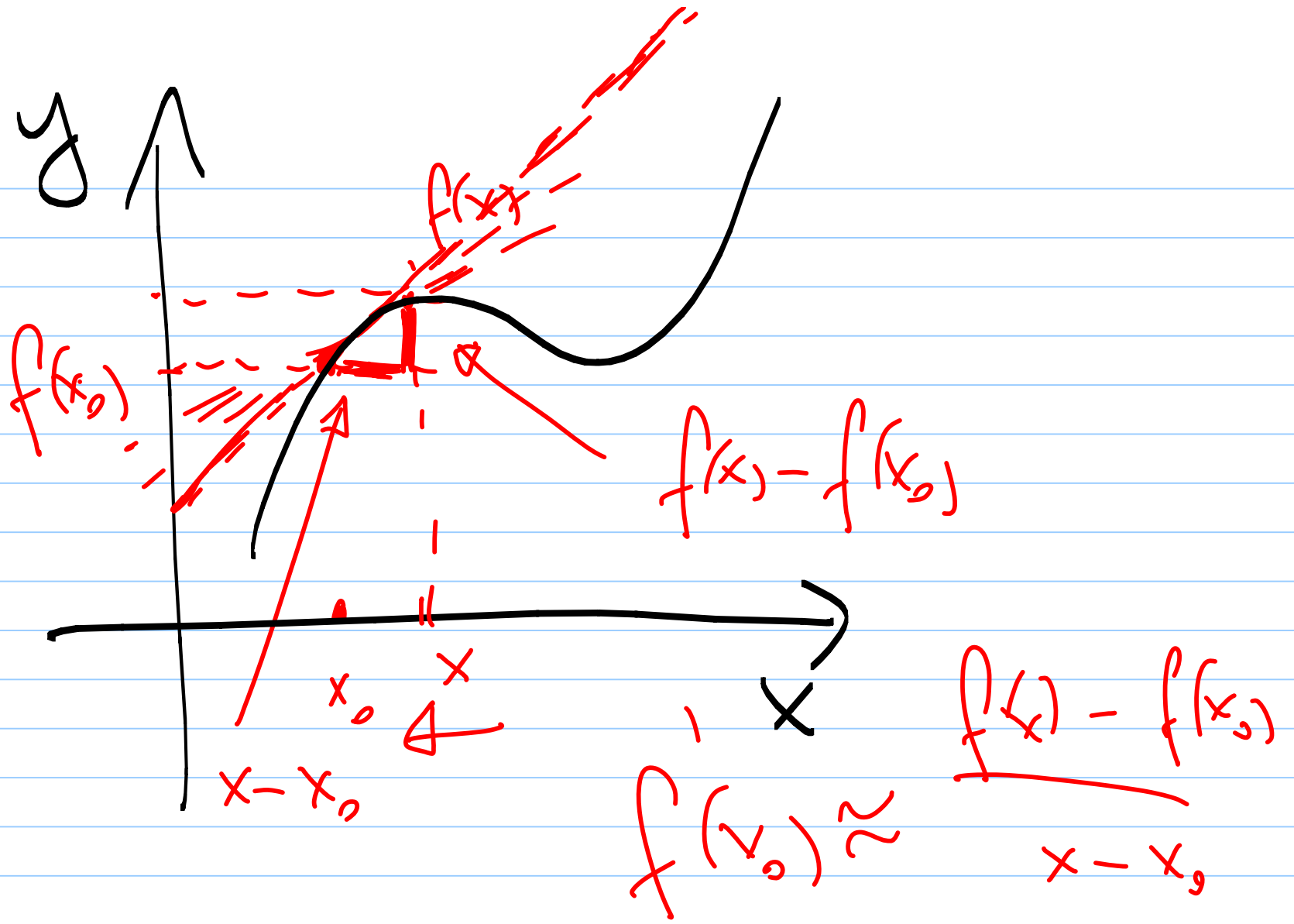
$$f(x) = (x^2 - 1)\sin\frac{2x-1}{x^2-1} \qquad x \neq \pm 1$$

ex. gjeje $f(x)$ i.e. per $x = \pm 1$?

$$\lim_{x \to 1} f(x) = 0$$

$$\left|\sin\frac{2x-1}{x^2-1}\right| \leq 1 \Rightarrow \quad -|x^2-1| \leq |f(x)| \leq |x^2-1|$$

$-|x^2-1| \to 0$ as $x \to \pm 1$; $|x^2-1| \to 0$ as $x \to \pm 1$.


y
f(x)
f(x_0)
f(x) - f(x_0)
x_0
x
x - x_0
f'(x_0) ≈
f(x) - f(x_0)
x - x_0