Nový nadpis 13.3.2013

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} = \underbrace{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + }_{\text{limitní}}$$

$$\sum_{k=1}^{\infty} a_k (-1)^{k+1} \leftarrow \underline{\text{ALTERNUJÍCÍ}}$$

konverguje $\Leftarrow$ $a_k \to 0$

$$\sum_{k=1}^{\infty} \frac{1}{k} = \infty$$

$$\left(\sum_{n=0}^{k} a_n\right)\cdot\left(\sum_{n=0}^{k} b_n\right) \longrightarrow \left(\sum_{n=0}^{\infty} a_n\right)\cdot\left(\sum_{n=0}^{\infty} b_n\right)$$

$$\sum_{0\leq n,m\leq k} a_n b_m \quad \times \quad \underbrace{\sum_{m=0}^{k}\sum_{i=0}^{m} a_i \cdot b_{m-i}}_{=:\, c_k}$$

$$\lim_{k\to\infty}\left(\sum_{n=0}^{k} a_n \cdot \sum_{n=0}^{k} b_n - \sum_{n=0}^{k} c_n\right) \overset{?}{=} 0$$

$$\left|\left(\sum_{n=0}^{k} a_n\right)\cdot\left(\sum_{n=0}^{k} b_n\right) - \sum_{n=0}^{k} c_n\right| = \left|\sum_{\substack{i+j>k \\ 0\le i,j\le k}} a_i b_j\right|$$

$$\le \sum_{\substack{i+j>k \\ 0\le i,j\le k}} |a_i b_j| \le \sum_{0\le i,j\le k} |a_i b_j| - \sum_{0\le i,j\le k/2} |a_i b_j|$$

$$\Big\downarrow k\to\infty \qquad\qquad \Big\downarrow k\to\infty$$

$$S\cdot T \qquad\qquad S\cdot T$$

7) NC pro $a_n \in \mathbb{R}$

$\lim\limits_{n\to\infty} a_n \neq 0$ (nebo neexistuje)

$\Rightarrow \exists\, \varepsilon > 0$ nekonečně mnoho $|a_n| > \varepsilon$.

$\Rightarrow$ mezi nimi ∞ mnoho buď >0 nebo <0

$\Rightarrow$ porušení Bolzano-Cauchyho podm. pro s_n, neboť $|s_n - s_{n-1}| > \varepsilon$.

$\Rightarrow \{s_n\}$ není konvergentní $\Rightarrow$ nekonverguje

② $0 \le q < r < 1$ $\qquad \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = q$

$\Rightarrow \exists N, \ \forall n > N \quad \boxed{a_{n+1} < r\, a_n}$

$\Rightarrow a_{n+1} < r\, a_n < r^{(n-N+1)} \cdot a_N$

$$s_n < \underbrace{\sum_{j=1}^{N} a_j}_{\text{[illegible]}} + a_N \underbrace{\sum_{j=0}^{n-N} r^j}_{= \frac{1 - r^{n-N+1}}{1-r}}$$

$r^{n-N+1} \to 0$

1) $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = q < r < 1$

$\exists N, \ \forall n > N \quad \sqrt[n]{a_n} < r$

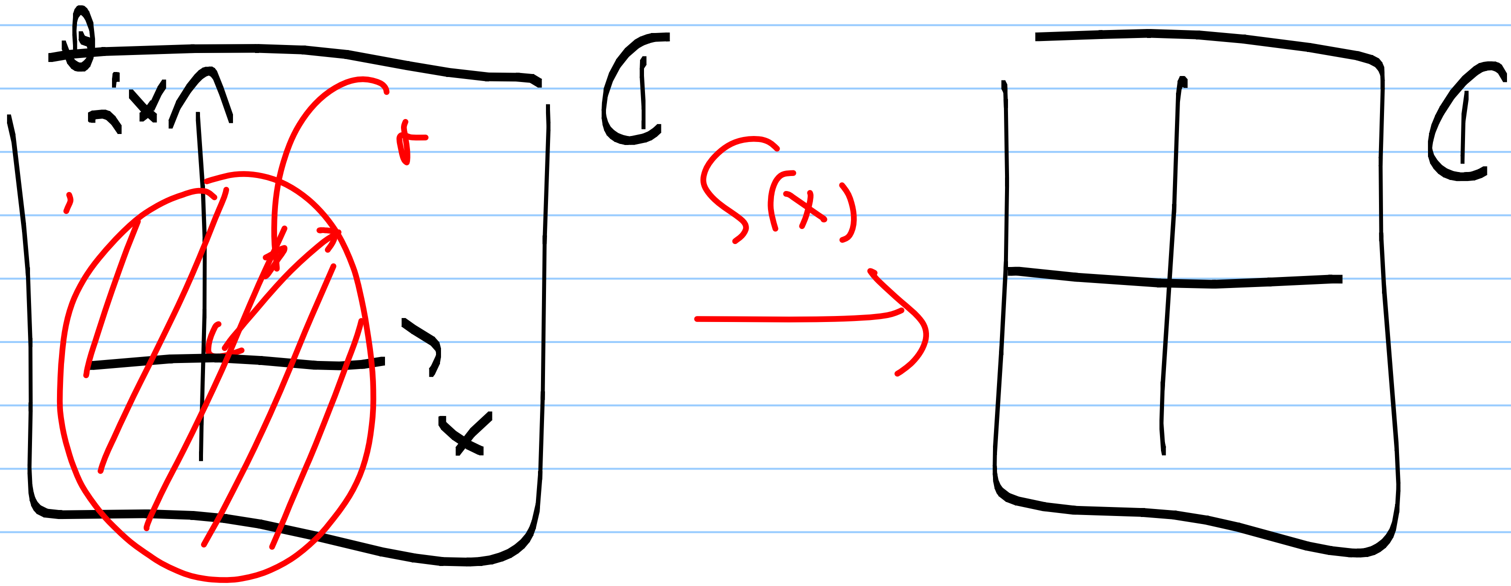
$\iff a_n < r^n$

$\Rightarrow$ opět srovnání s geometrickou řadou.

a podobně pro $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = q > r > 1 \Rightarrow a_n > r^n$

$$S(x) = \sum_{n=0}^{\infty} a_n x^n \qquad S: \mathbb{C} \to \mathbb{C}$$

$\mathcal{D}$ TAM, KDE KONVERGUJE



$$S(x) = \sum_{n=0}^{\infty} \boxed{a_n x^n}$$

$$\limsup_{n\to\infty} \sqrt[n]{|a_n||x|^n} = \limsup_{n\to\infty} \sqrt[n]{|a_n|}\cdot|x|$$

$$\Rightarrow \text{wo } |x|\cdot\limsup_{n\to\infty}\sqrt[n]{|a_n|} < 1 \quad \text{konvergiert}$$

$$> 1 \quad \text{divergiert}$$

2^{k-1} ks čísel jejichž

odhadnu 2^{-k}

$\Rightarrow 2^{k-1}/2^{k} = \frac{1}{2}$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$

$\frac{1}{2^{k-1}}, \dots, \frac{1}{(2^{k}-1)}$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \dots + \frac{1}{n!}x^n + \dots$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{1}{(n+1)!}x^{n+1}}{\frac{1}{n!}x^n} = \frac{x}{n+1} \to 0 \checkmark$$

$$\sqrt[n]{a_n} = \sqrt[n]{\frac{x^n}{n!}} = x \cdot \sqrt[n]{\frac{1}{n!}} \to ?$$