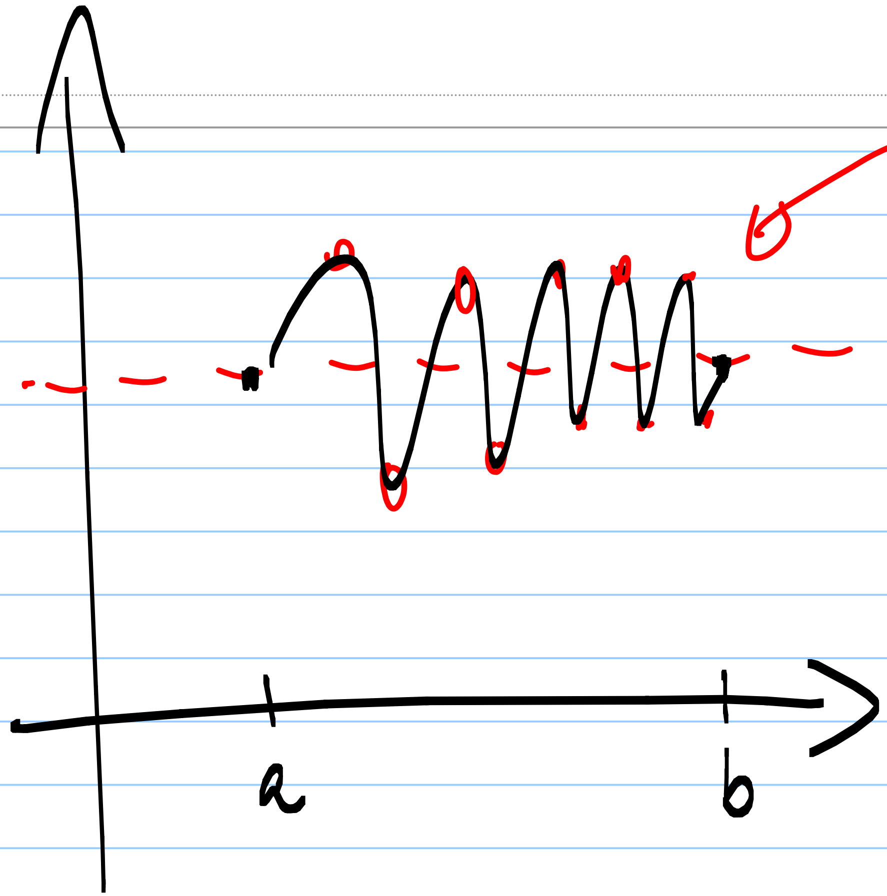
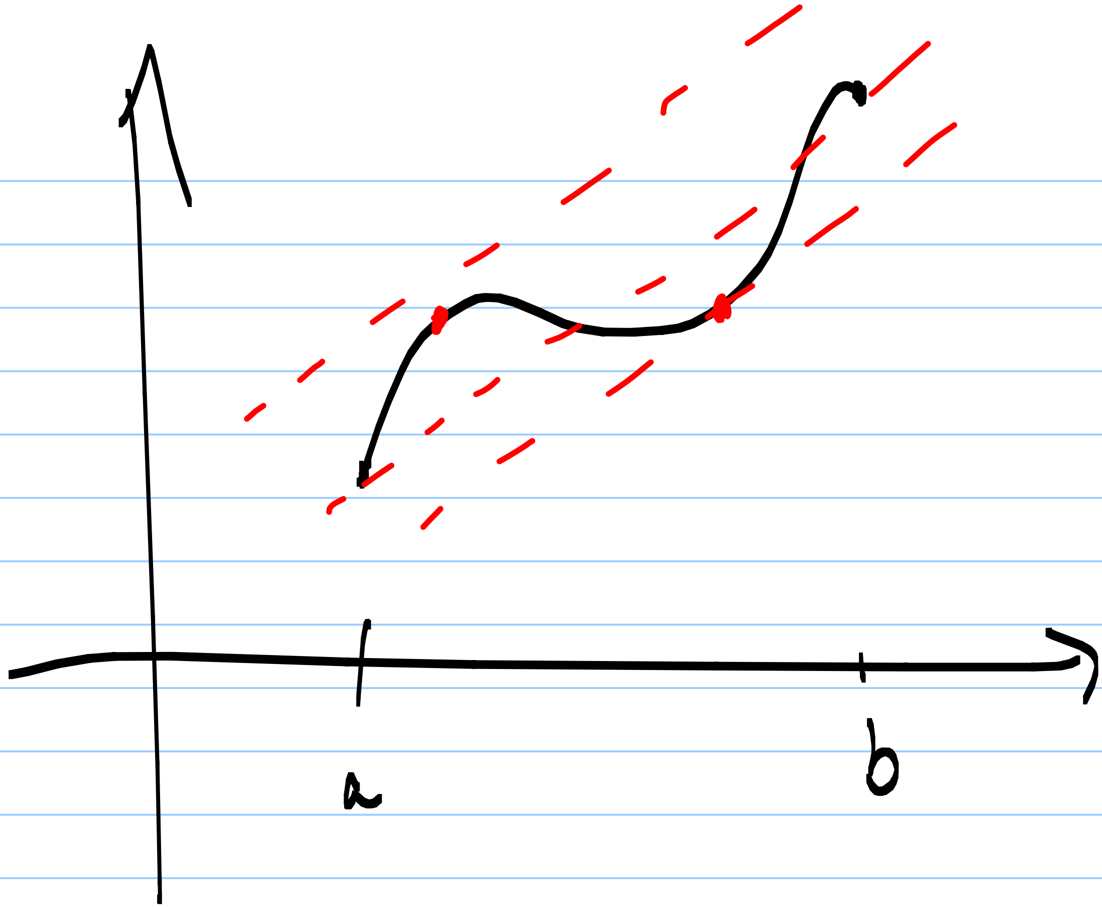
Nový nadpis 11.3.2013

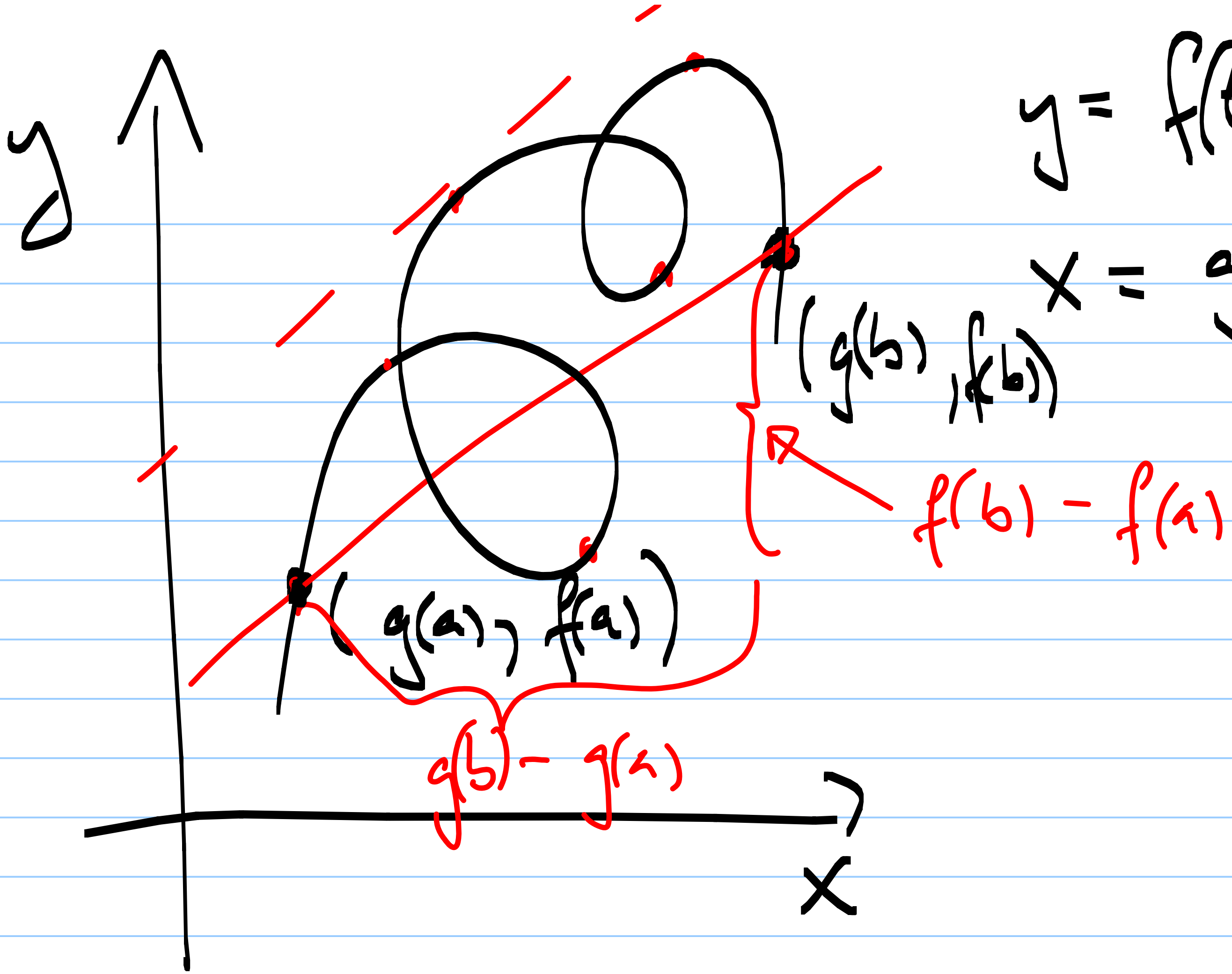
DERIVACE 0

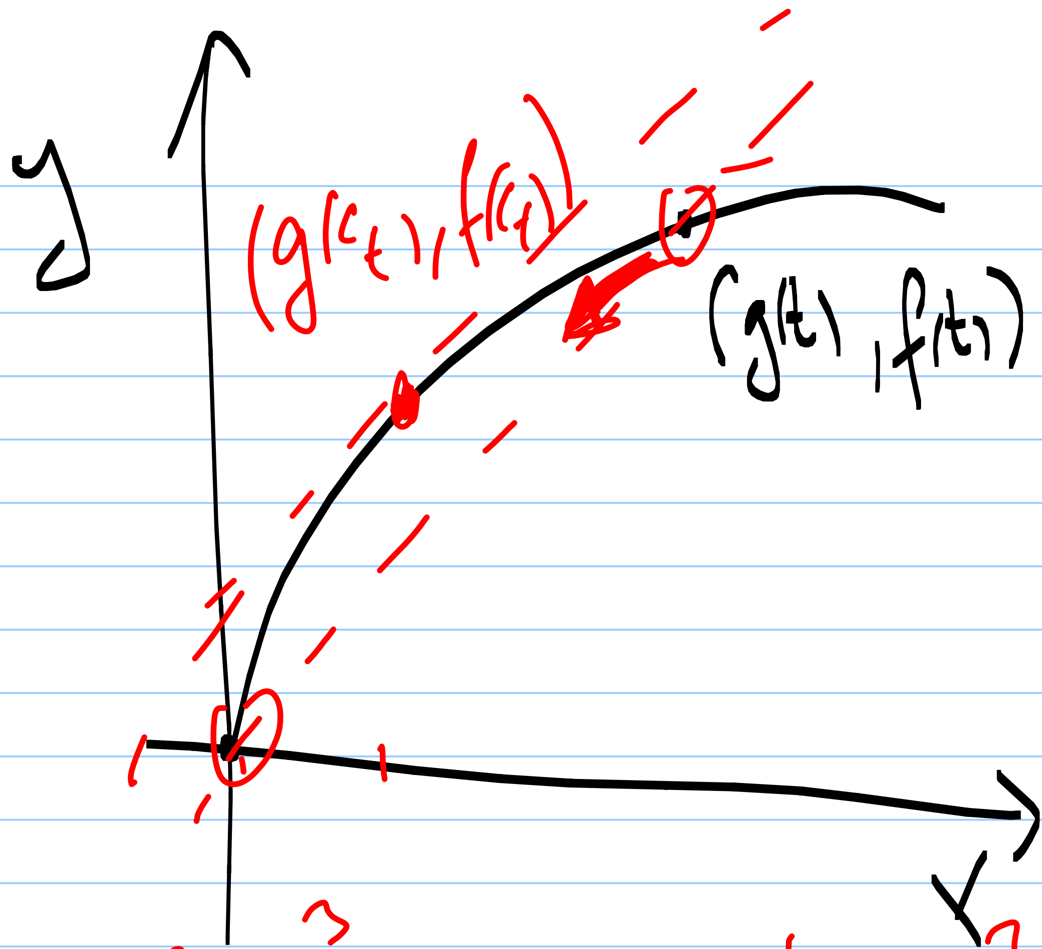



a
b

$y = f(t)$

$x = g(t)$





$x = g(t) \qquad g(0) = 0$

$y = f(t) \qquad f(0) = 0$

$$= \frac{2}{3}$$

$$\lim_{x\to 0} \frac{2x^2 + x^3}{3x^2 + 13x^4} = \lim_{x\to 0} \frac{4x + 3x^2}{6x + 52x^3} = \lim_{x\to 0} \frac{4 + 6x}{6 + 156x^2}$$

$$\left(1+\frac{x}{12}\right)^{12} \qquad \left(1+\frac{x}{365}\right)^{365} \qquad \left(1+\frac{x}{8760}\right)^{8760}$$

$$\boxed{a_n = \left(1+\frac{1}{n}\right)^n}$$

$$\Downarrow$$

$$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots + \frac{1}{k!}x^k + \ldots$$

Bernoulli: $\forall b \in \mathbb{R},\ b \geq -1,\ b \neq 0$

$n \geq 2 \quad (1+b)^n > 1+nb$

DL: $n=2$: $(1+b)^2 = 1+2b+b^2 > 1+2b$ ✓

induzione: $(1+b)^{k+1} = \underbrace{(1+b)^k}(1+b) > (1+nb)(1+b)$

$= 1+nb+b+nb^2$

$> 1+(n+1)b$

7. $a_n = \left(1+\frac{1}{n}\right)^n$ je rostoucí.

$$\frac{a_n}{a_{n-1}} = \frac{\left(1+\frac{1}{n}\right)^n}{\left(1+\frac{1}{n-1}\right)^{n-1}} = \frac{(n^2-1)^n\, n}{n^{2n}(n-1)} = \left(1-\frac{1}{n^2}\right)^n \cdot \frac{n}{(n-1)}$$

$$> \left(1-\frac{1}{n}\right)\frac{n}{n-1} = 1$$

$$b_n = \left(1+\frac{1}{n}\right)^{n+1} > a_n$$

b_n je klesající

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)a_n$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = e$$

$$\lim_{\Delta x \to 0} \frac{e^{\Delta x} - e^{0}}{\Delta x} = \frac{e^{\Delta x} - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1 + 1 + \Delta x + \frac{\Delta x^2}{2!} + \dots}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(1 + \cancel{\frac{\Delta x}{2!} + \frac{\Delta x^2}{3!} + \dots} \right)$$

$$= \underline{\underline{1}}$$